(10/1/08)

Math 20C. Lecture Examples.

Sections 11.1 and 13.1. Parametric equations and vector-valued functions[†]

Example 1 **Draw the curve C: $\mathbf{x = x(t), y = y(t), 0 \le t \le 3}$ in an xy-plane, where $\mathbf{x = x(t)}$ and $\mathbf{y = y(t)}$ are the piecewise-linear functions whose graphs are in Figures 1 and 2.**

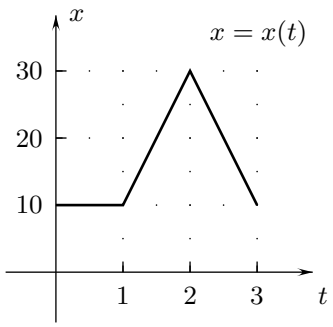


FIGURE 1

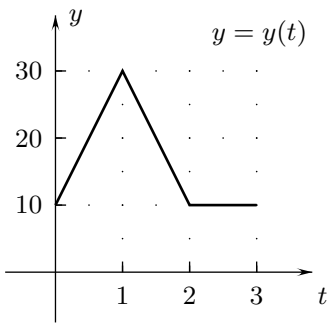


FIGURE 2

Answer: Use the values in the table below. ● Figure A1

t	0	1	2	3
$x = x(t)$	10	10	30	10
$y = y(t)$	10	30	10	10

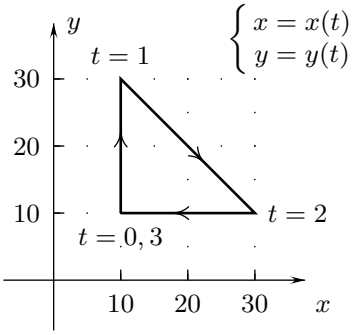


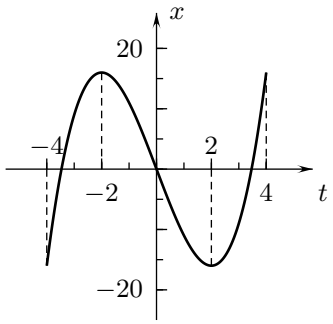
Figure A1

[†]Lecture notes to accompany Sections 11.1 and 13.1 of *Calculus, Early Transcendentals* by Rogawski.

Example 2 **Sketch the curve C: $\mathbf{x = t^3 - 12t, y = 2t^2, -4 \le t \le 4}$.**

Answer: Draw $x = x(t)$ in a tx-plane and $y = y(t)$ in a ty-plane for $-4 \le t \le 4$, using the table of values below. • Figures A2a and A2b). • Use these graphs to draw C: $x = t^3 - 12t, y = 2t^2, -4 \le t \le 4$ in an xy-plane. • Figure A2c

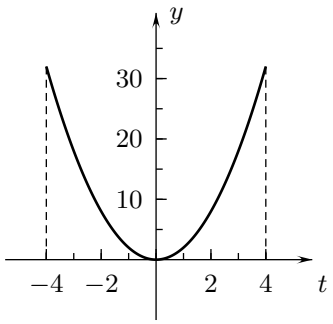
t	-4	-2	0	2	4
$x = t^3 - 12t$	-16	16	0	-16	16
$y = 2t^2$	32	8	0	8	32



$x(t) = t^3 - 12t$

$-4 \le t \le t$

Figure A2a



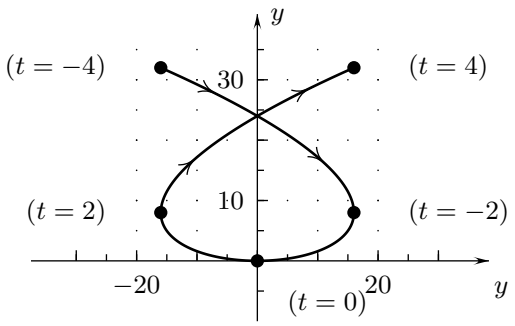
$y(t) = 2t^2$

$-4 \le t \le 4$

Figure A2b

$$\begin{cases} x = t^3 - 12t \\ y = 2t^2 \\ -4 \le t \le 4 \end{cases}$$

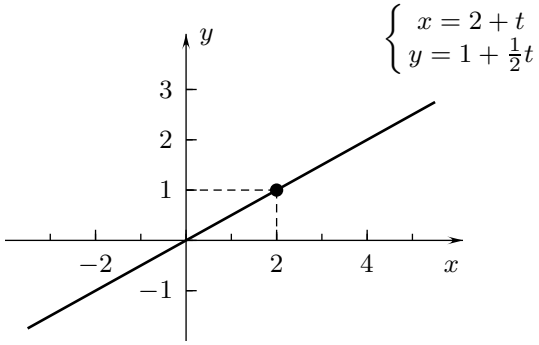
Figure A2c



Example 3 **Draw the line with parametric equations $\mathbf{x = 2 + t, y = 1 + \frac{1}{2}t}$.**

Answer: Figure A3

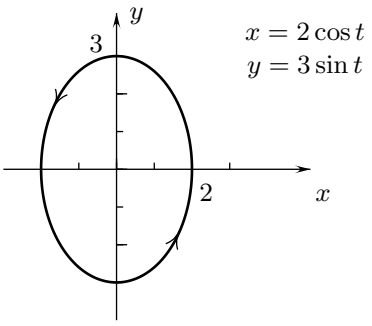
Figure A3



Example 4 **Draw the ellipse C: $\mathbf{x = 2\cos t, y = 3\sin t, 0 \le t \le 2\pi}$.**

Answer: Figure A4

Figure A4



Example 5 **When the circle of radius 1 in Figure 3 is rolled to the right along the x-axis, the point P that is initially at the origin generates the *cycloid* in Figure 4. Give parametric equations of this curve.**

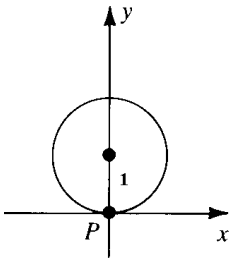


FIGURE 3

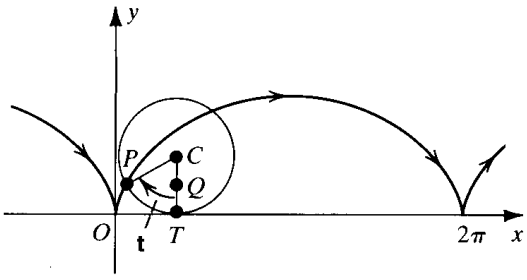


FIGURE 4

Answer: $x = t - \sin t, y = 1 - \cos t$

Example 6 **What is the slope of the tangent line at $\mathbf{t = 3}$ on the curve $\mathbf{x = t^3 - 12t, y = 2t^2}$ in Figure 5?**

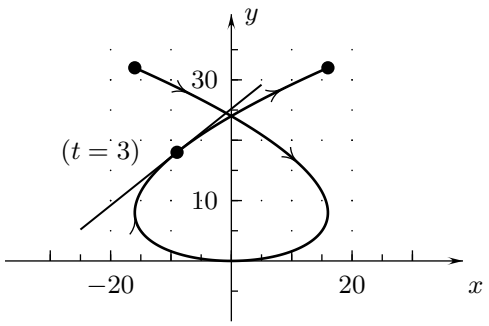


FIGURE 5

Answer: [Slope of the tangent line at $t = 3$] $= \frac{4}{5}$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 13.1: Examples 1–5

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.