Math 20C. Lecture Examples.

Sections 11.2, 13.2, and 13.3. Calculus of vector-valued functions†

Example 1 Find \( \lim_{t \to 1} \langle t^2 - 3, e^{3t}, \ln t \rangle. \)
Answer: \( \lim_{t \to 1} \langle t^2 - 3, e^{3t}, \ln t \rangle = \langle -2, e^3, 0 \rangle \)

Example 2 What is \( \lim_{t \to 3} r(t) \) if \( r(t) = \langle -t, t^2 - 5 \rangle \)?
Answer: \( \lim_{t \to 3} r(t) = \langle -3, 4 \rangle \)

Example 3 Find the derivative, \( \frac{d}{dt} \langle t^2 - 3, e^{3t}, \ln t \rangle. \)
Answer: \( \frac{d}{dt} \langle t^2 - 3, e^{3t}, \ln t \rangle = \langle 2t, 3e^{3t}, \frac{1}{t} \rangle \)

Example 4 What is the derivative \( r'(\frac{1}{3}\pi) \) for \( r(t) = 2 \cos t \mathbf{i} + 4 \sin t \mathbf{j} \)?
Answer: \( r'(\frac{1}{3}\pi) = -\sqrt{3}\mathbf{i} + 2\mathbf{j} \)

Example 5 Find the velocity vector to the ellipse \( C: x = 5 \cos t, y = 3 \sin t \) at \( t = \frac{1}{4}\pi \). Then draw the ellipse and the velocity vector, using the scales on the axes to measure the components.
Answer: \( v(\frac{1}{4}\pi) = \langle -\frac{5}{2}\sqrt{2}, \frac{3}{2}\sqrt{2} \rangle \) • Figures A5a and A5b

![Figure A5a](image1)
![Figure A5b](image2)

†Lecture notes to accompany Sections 11.2, 13.2, and 13.3 of Calculus, Early Transcendentals by Rogawski.
Example 6  Figure 1 shows the curve \( C: x = 4t - t^3, y = 2 - \sin(\pi t), \ -1 \leq t \leq 2.3. \)

(a) Find its velocity vector at \( t = 1 \) and draw it with the curve using the scales on the axes to measure the components.  
(b) What is the speed at \( t = 1 \)?

Answer: (a)  Velocity at \( t = 1 \) = \( \langle 1, \pi \rangle \) \( \approx \langle 1, 3.14 \rangle \).

• Put the base of the velocity vector at \( (x(1), y(1)) = (3, 2) \).

• Figure A6  (b)  Speed at \( t = 1 \) = \( \sqrt{1 + \pi^2} \).
Example 7  A robot moving in an xy-plane with distances measured in meters is at (120,40) at \( t = 0 \) (minutes) and its velocity vector is \( \mathbf{v}(t) = (-120 \sin(2t), 80 \cos(2t)) \) (meters per minute) at time \( t \). Find the robot’s position vector \( \mathbf{R} = \mathbf{R}(t) \) and describe the robot’s path.

**Answer:** \( \mathbf{R}(t) = \langle 60 \cos(2t) + 60, 40 \sin(2t) + 40 \rangle \) (meters)  
- The path is the ellipse in Figure A7 with center at (60, 40), horizontal axis of length 120, and vertical axis of length 80.

![Figure A7](image)

Example 8  Find the length of the curve \( x = t^3, y = t^2, -1 \leq t \leq 1 \) in Figure 2.

\[
\begin{align*}
\begin{cases}
x = t^3 \\
y = t^2 \\
-1 \leq t \leq 1
\end{cases}
\end{align*}
\]

**FIGURE 2**

**Answer:** \( \text{[Length]} = \frac{2}{3} (13^{3/2} - 8) \)
Example 9  
Give a definite integral that equals the length of the curve, 
C: \( x = 2 \sin t, \quad y = \ln t, \quad 0.1 \leq t \leq 10 \) in Figure 3.

\[
\left\{ \begin{array}{l}
  x = 2 \sin t \\
  y = \ln t \\
  0.1 \leq t \leq 10
\end{array} \right.
\]

FIGURE 3

**Answer:** 
\[
\text{[Length]} = \int_{0.1}^{10} \sqrt{4 \cos^2 t + t^{-2}} \, dt
\]

Interactive Examples

Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/:

†Section 13.2: Examples 1–5

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†The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.