

Math 20C. Lecture Examples.

Sections 11.2, 13.2, and 13.3. Calculus of vector-valued functions[†]

Example 1 Find $\lim_{t \rightarrow 1} \langle t^2 - 3, e^{3t}, \ln t \rangle$.

Answer: $\lim_{t \rightarrow 1} \langle t^2 - 3, e^{3t}, \ln t \rangle = \langle -2, e^3, 0 \rangle$

Example 2 What is $\lim_{t \rightarrow 3} \mathbf{r}(t)$ if $\mathbf{r}(t) = \langle -t, t^2 - 5 \rangle$?

Answer: $\lim_{t \rightarrow 3} \mathbf{r}(t) = \langle -3, 4 \rangle$

Example 3 Find the derivative, $\frac{d}{dt} \langle t^2 - 3, e^{3t}, \ln t \rangle$.

Answer: $\frac{d}{dt} \langle t^2 - 3, e^{3t}, \ln t \rangle = \langle 2t, 3e^{3t}, \frac{1}{t} \rangle$

Example 4 What is the derivative $\mathbf{r}'(\frac{1}{3}\pi)$ for $\mathbf{r}(t) = 2 \cos t \mathbf{i} + 4 \sin t \mathbf{j}$?

Answer: $\mathbf{r}'(\frac{1}{3}\pi) = -\sqrt{3}\mathbf{i} + 2\mathbf{j}$

Example 5 Find the velocity vector to the ellipse $\mathbf{C}: x = 5 \cos t, y = 3 \sin t$ at $t = \frac{1}{4}\pi$. Then draw the ellipse and the velocity vector, using the scales on the axes to measure the components.

Answer: $\mathbf{v}(\frac{1}{4}\pi) = \langle -\frac{5}{2}\sqrt{2}, \frac{3}{2}\sqrt{2} \rangle$ • Figures A5a and A5b

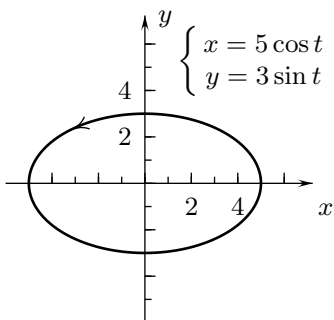


Figure A5a

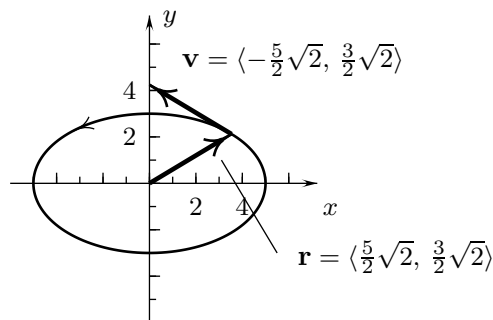
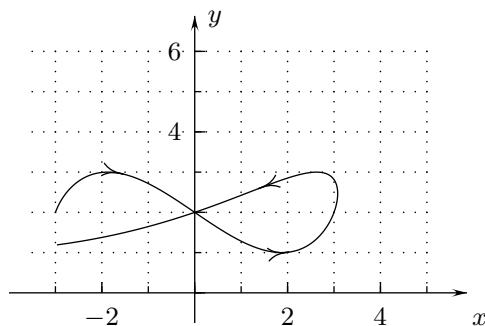


Figure A5b

[†]Lecture notes to accompany Sections 11.2, 13.2, and 13.3 of *Calculus, Early Transcendentals* by Rogawski.

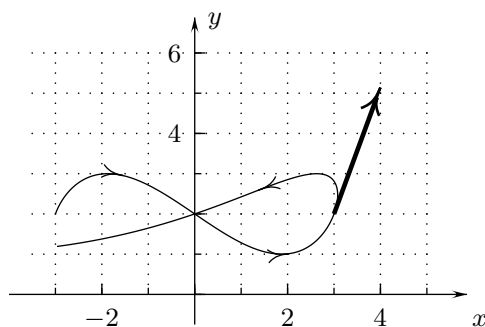
Example 6 Figure 1 shows the curve $C: x = 4t - t^3, y = 2 - \sin(\pi t), -1 \leq t \leq 2.3$.
 (a) Find its velocity vector at $t = 1$ and draw it with the curve using the scales on the axes to measure the components. (b) What is the speed at $t = 1$?

FIGURE 1



Answer: (a) [Velocity at $t = 1$] = $\langle 1, \pi \rangle \doteq \langle 1, 3.14 \rangle$ • Put the base of the velocity vector at $(x(1), y(1)) = (3, 2)$.
 • Figure A6 (b) [Speed at $t = 1$] = $\sqrt{1 + \pi^2}$

Figure A6



Example 7 A robot moving in an xy -plane with distances measured in meters is at $(120,40)$ at $t = 0$ (minutes) and its velocity vector is $\mathbf{v}(t) = \langle -120 \sin(2t), 80 \cos(2t) \rangle$ (meters per minute) at time t . Find the robot's position vector $\mathbf{R} = \mathbf{R}(t)$ and describe the robot's path.

Answer: $\mathbf{R}(t) = \langle 60 \cos(2t) + 60, 40 \sin(2t) + 40 \rangle$ (meters) • The path is the ellipse in Figure A7 with center at $(60, 40)$, horizontal axis of length 120, and vertical axis of length 80.

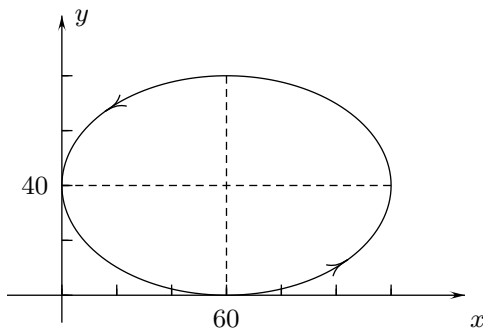
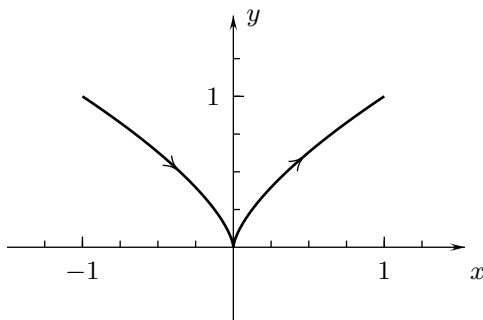


Figure A7

Example 8 Find the length of the curve $x = t^3, y = t^2, -1 \leq t \leq 1$ in Figure 2.

$$\begin{cases} x = t^3 \\ y = t^2 \\ -1 \leq t \leq 1 \end{cases}$$

FIGURE 2

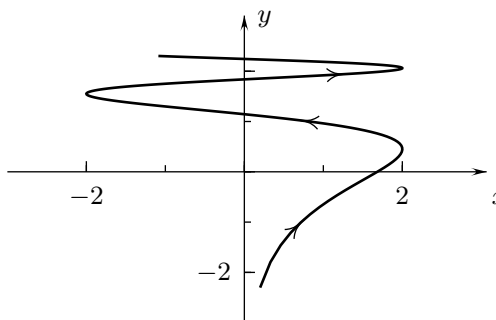


Answer: $[\text{Length}] = \frac{2}{27}(13^{3/2} - 8)$

Example 9 Give a definite integral that equals the length of the curve,
C: $x = 2 \sin t$, $y = \ln t$, $0.1 \leq t \leq 10$ in Figure 3.

$$\begin{cases} x = 2 \sin t \\ y = \ln t \\ 0.1 \leq t \leq 10 \end{cases}$$

FIGURE 3



Answer: [Length] = $\int_{0.1}^{10} \sqrt{4 \cos^2 t + t^{-2}} dt$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, <http://www.math.ucsd.edu/~ashenk/>.[‡]

Section 13.2: Examples 1–5

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.