Math 20C. Lecture Examples.

Sections 11.2, 13.2, and 13.3. Calculus of vector-valued functions†

Example 1 **Find $\lim\limits_{t\to 1} \langle t^2 - 3,\ e^{3t},\ \ln t\rangle$.**

Answer: $\lim\limits_{t\to 1} \langle t^2 - 3,\ e^{3t},\ \ln t\rangle = \langle -2,\ e^3,\ 0\rangle$

Example 2 **What is $\lim\limits_{t\to 3} \mathbf{r}(t)$ if $\mathbf{r}(t) = \langle -t, t^2 - 5\rangle$?**

Answer: $\lim\limits_{t\to 3} \mathbf{r}(t) = \langle -3, 4\rangle$

Example 3 **Find the derivative, $\dfrac{d}{dt}\langle t^2 - 3,\ e^{3t},\ \ln t\rangle$.**

Answer: $\dfrac{d}{dt}\langle t^2 - 3,\ e^{3t},\ \ln t\rangle = \left\langle 2t,\ 3e^{3t},\ \dfrac{1}{t}\right\rangle$

Example 4 **What is the derivative $\mathbf{r}'(\frac{1}{3}\pi)$ for $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j}$?**

Answer: $\mathbf{r}'(\frac{1}{3}\pi) = -\sqrt{3}\mathbf{i} + 2\mathbf{j}$

Example 5 **Find the velocity vector to the ellipse C: $= 5\cos t, y = 3\sin t$ at $t = \frac{1}{4}\pi$. Then draw the ellipse and the velocity vector, using the scales on the axes to measure the components.**

Answer: $\mathbf{v}(\frac{1}{4}\pi) = \langle -\frac{5}{2}\sqrt{2}, \frac{3}{2}\sqrt{2}\rangle$ • Figures A5a and A5b

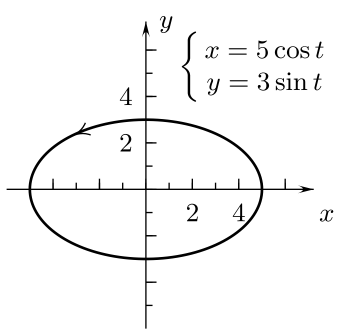


Figure A5a

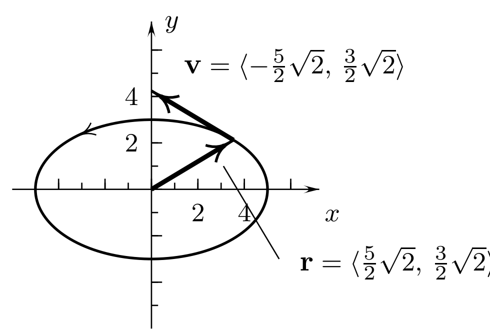
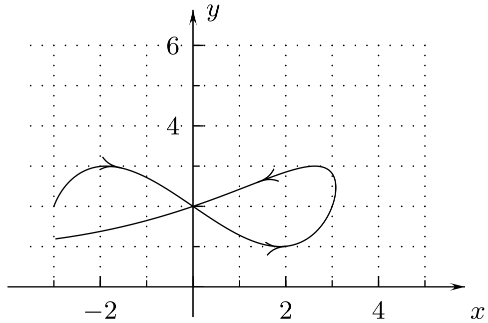


Figure A5b

†Lecture notes to accompany Sections 11.2, 13.2, and 13.3 of *Calculus, Early Transcendentals* by Rogawski.

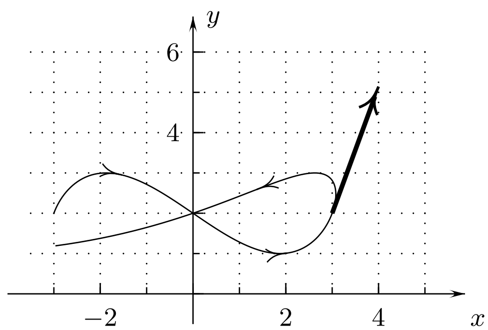
Example 6 **Figure 1 shows the curve C: $\mathbf{x = 4t - t^3, y = 2 - \sin(\pi t), -1 \le t \le 2.3}$. (a) Find its velocity vector at $\mathbf{t = 1}$ and draw it with the curve using the scales on the axes to measure the components. (b) What is the speed at $\mathbf{t = 1}$?**

FIGURE 1



Answer: (a) [Velocity at $t = 1$] $= \langle 1, \pi \rangle \doteq \langle 1, 3.14 \rangle$ • Put the base of the velocity vector at $(x(1), y(1)) = (3, 2)$. • Figure A6 **(b)** [Speed at $t = 1$] $= \sqrt{1 + \pi^2}$

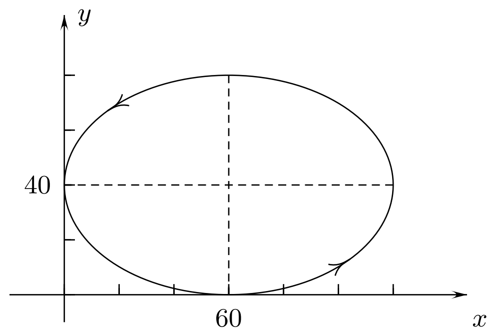
Figure A6



Example 7 **A robot moving in an xy-plane with distances measured in meters is at (120,40) at $\mathbf{t = 0}$ (minutes) and its velocity vector is $\mathbf{v(t) = \langle -120\sin(2t),\ 80\cos(2t)\rangle}$ (meters per minute) at time $\mathbf{t}$. Find the robot's position vector $\mathbf{R = R(t)}$ and describe the robot's path.**

Answer: $\mathbf{R}(t) = \langle 60\cos(2t) + 60,\ 40\sin(2t) + 40\rangle$ (meters) • The path is the ellipse in Figure A7 with center at $(60, 40)$, horizontal axis of length 120, and vertical axis of length 80.

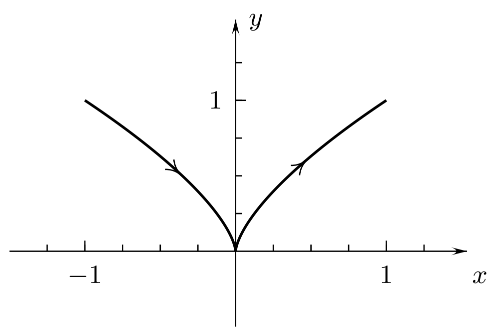
Figure A7



Example 8 **Find the length of the curve $\mathbf{x = t^3, y = t^2, -1 \le t \le 1}$ in Figure 2.**

$$\begin{cases} x = t^3 \\ y = t^2 \\ -1 \le t \le 1 \end{cases}$$

FIGURE 2

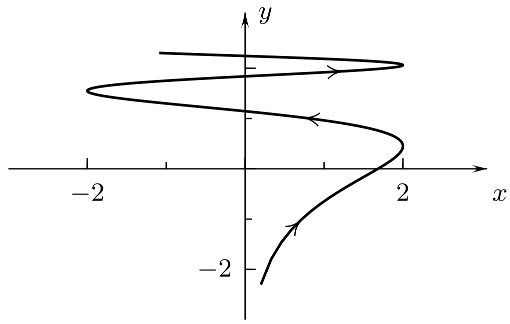


Answer: [Length] $= \frac{2}{27}(13^{3/2} - 8)$

Example 9 **Give a definite integral that equals the length of the curve, C: $\mathbf{x = 2\sin t,\ y = \ln t,\ 0.1 \le t \le 10}$ in Figure 3.**

$$\begin{cases} x = 2\sin t \\ y = \ln t \\ 0.1 \le t \le 10 \end{cases}$$

FIGURE 3



Answer: $[\text{Length}] = \int_{0.1}^{10} \sqrt{4\cos^2 t + t^{-2}}\, dt$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 13.2: Examples 1–5

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.