(10/1/08)

Math 20C. Lecture Examples.

Sections 14.2 and 14.3. Limits and partial derivatives†

Example 1 **What is $\lim_{(\mathbf{x},\mathbf{y})\to(\mathbf{3},\mathbf{2})}(\mathbf{x}^2+\mathbf{y}^2)$?**

Answer: $\lim_{(x,y)\to(3,2)}(x^2+y^2)=13$

Example 2 **Find the x- and y-derivatives of $\mathbf{f}(\mathbf{x},\mathbf{y})=\mathbf{x}^3\mathbf{y}-\mathbf{x}^2\mathbf{y}^5+\mathbf{x}$.**

Answer: $\dfrac{\partial f}{\partial x}=3x^2y-2xy^5+1$ • $\dfrac{\partial f}{\partial y}=x^3-5x^2y^4$

Example 3 **What are $\mathbf{g_x}(\mathbf{2},\mathbf{5})$ and $\mathbf{g_y}(\mathbf{2},\mathbf{5})$ for $\mathbf{g}(\mathbf{x},\mathbf{y})=\mathbf{x}^2\mathbf{e}^{3\mathbf{y}}$?**

Answer: $g_x(2,5)=4e^{15}$ • $g_y(2,5)=12e^{15}$

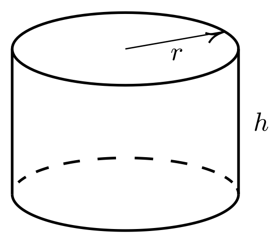
Example 4 **The volume of a right circular cylinder of radius r and height h is equal to the product $\mathbf{V}(\mathbf{r},\mathbf{h})=\pi\mathbf{r}^2\mathbf{h}$ of its height h and the area $\pi\mathbf{r}^2$ of its base (Figure 1). What are (a) the rate of change of the volume with respect to the radius and (b) the rate of change of the volume with respect to the height and what are their geometric significance?**

[Area of base] $=\pi r^2$

[Volume] $=\pi r^2 h$

[Lateral surface area] $=2\pi rh$

FIGURE 1



Answer: (a) $\dfrac{\partial V}{\partial r}=2\pi rh$ is the area of the lateral surface (the sides) of the cylinder.

(b) $\dfrac{\partial V}{\partial h}=\pi r^2$ is the area of the base.

†Lecture notes to accompany Sections 14.2 and 14.3 of *Calculus, Early Transcendentals* by Rogawski.

Example 5 **The table below is from a study of the effect of exercise on the blood pressure of women. $\mathbf{P} = \mathbf{P}(\mathbf{t}, \mathbf{E})$ is the average blood pressure, measured in millimeters of mercury (mm Hg), of women of age $\mathbf{t}$ years who are exercising at the rate of $\mathbf{E}$ watts.[1] (One watt is 0.86 Calories per hour.) What is the approximate rate of change with respect to age of the average blood pressure of forty-five-year old women who are exercising at the rate of 100 watts?**

$\mathbf{P} = \mathbf{P}(\mathbf{t}, \mathbf{E})$ (millimeters of mercury)

	t = 25	t = 35	t = 45	t = 55	t = 65
E = 150	178	180	197	209	195
E = 100	163	165	181	199	200
E = 50	145	149	167	177	181
E = 0	122	125	132	140	158

Answer: $P_t(45, 100) \approx 1.8$ millimeters of mercury per year (using a right difference quotient); or $P_t(45, 100) \approx 1.6$ millimeters of mercury per year (using a left difference quotient); or $P_t(45, 100) \approx 1.7$ millimeters of mercury per year (using a centered difference quotient)

Example 6 **Use the table from Example 5 to find the approximate rate of change with respect to age of the average blood pressure of fifty-five-year-old women who are exercising at the rate of 75 watts.**

Answer: $\left.\dfrac{\partial P}{\partial E}\right|_{(62,75)} \approx 0.44$ millimeters of mercury per watt

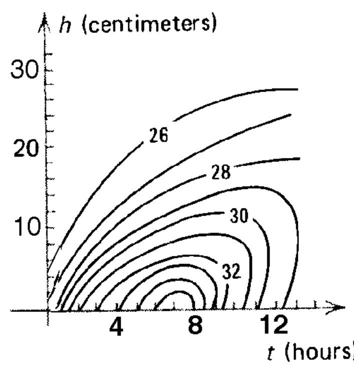
[1] Data adapted from *Geigy Scientific Tables*, edited by C. Lentner, Vol. 5, Basel, Switzerland: CIBA-GEIGY Limited, 1990, p. 29.

Example 7 **Figure 2 shows level curves of the temperature $\mathbf{T} = \mathbf{T}(\mathbf{t}, \mathbf{h})^\circ\mathbf{F}$ as a function of time t (hours) and the depth h (centimeters) beneath the surface of the ground at O'Neil, Nebraska, from noon one day ($t = 0$) until the next morning.**[2]

(a) What was the approximate rate of change of the temperature with respect to time at 4:00 PM at a point 14 centimeters beneath the surface of the ground?

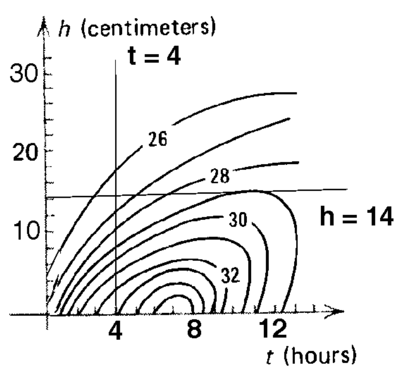
(b) What was the approximate rate of change of the temperature with respect to depth at 4:00 PM at a point 14 centimeters beneath the surface of the ground?

FIGURE 2



Answer: (a) Figure A7 • $T_t(4, 14) \approx 0.5$ degree per hour **(b)** $T_h(4, 14) \approx -0.25$ degree per centimeter

Figure A7



[2] Data adapted from *Fundamentals of Air Pollution* by S. Williamson, Reading, MA: Addison Wesley, 1973.

Example 8 **What are the first-order partial derivatives of $\mathbf{f = x^2y^3z^4}$?**

Answer: $f_x = 2xy^3z^4$ • $f_y = 3x^2y^2z^4$ • $f_z = 4x^2y^3z^3$

Example 9 **What are (a) $\mathbf{h_{yz}}$ and (b) $\mathbf{h_{zy}}$ for $\mathbf{h(x,y,z) = e^x \sin y \cos z}$?**

Answer: **(a)** $h_{yz} = -e^x \cos y \sin z$ **(b)** $h_{zy} = -e^x \cos y \sin z$

Example 10 **Find the fourth derivative $\dfrac{\partial^4}{\partial w \partial x \partial y \partial z}(w^2x^2y^2z^2)$.**

Answer: $\dfrac{\partial^4}{\partial w \partial x \partial y \partial z}(w^2x^2y^2z^2) = 16wxyz$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:‡

Section 14.3: Examples 1 through 5

Section 14.7: Example 2

Section 14.8: Example 2

‡The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.