(10/8/08)

Math 20C. Lecture Examples.

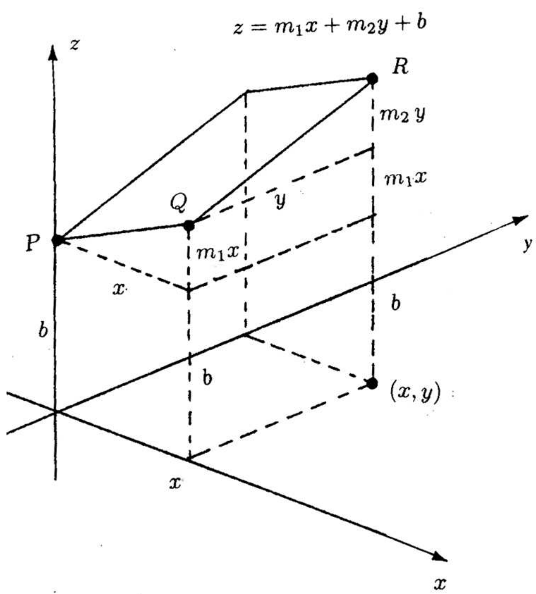
Section 14.4. Linear approximations and tangent planes[†]

Theorem (a) (The slope-intercept equation of a plane) ***Suppose that the*** $\mathbf{z}$***-intercept of a plane is*** $\mathbf{b}$***, the slope of its vertical cross sections in the positive*** $\mathbf{x}$***-direction is*** $\mathbf{m_1}$***, and the slope of its vertical cross sections in the positive*** $\mathbf{y}$***-direction is*** $\mathbf{m_2}$ ***(Figure 1). Then the plane has the equation,***

$$\mathbf{z = m_1 x + m_2 y + b.} \tag{1}$$

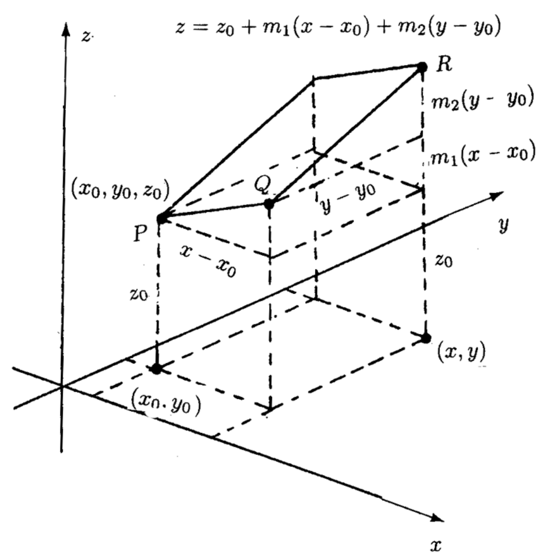
(b) (The point-slope equation of a plane) ***Suppose that a plane contains the point*** $(\mathbf{x_0, y_0, z_0})$***, the slope of its vertical cross sections in the positive*** $\mathbf{x}$***-direction is*** $\mathbf{m_1}$***, and the slope of its vertical cross sections in the positive*** $\mathbf{y}$***-direction is*** $\mathbf{m_2}$ ***(Figure 2). Then the plane has the equation,***

$$\mathbf{z = z_0 + m_1(x - x_0) + m_2(y - y_0).} \tag{2}$$



The slope-intercept equation

FIGURE 1



The point-slope equation

FIGURE 2

Example 1 **Give an equation of the plane with slope -6 in the positive $\mathbf{x}$-direction, slope 7 in the positive $\mathbf{y}$-direction, and $\mathbf{z}$-intercept 10.**

Answer: $z = -6x + 7y + 10$

Example 2 **Give an equation of the plane through the point (1,2,3) with slope 4 in the positive $\mathbf{x}$-direction and slope -5 in the positive $\mathbf{y}$-direction.**

Answer: $z = 3 + 4(x - 1) - 5(y - 2)$

[†]Lecture notes to accompany Section 14.4 of *Calculus, Early Transcendentals* by Rogawski.

Example 3 **Find a formula for the linear function $\mathbf{z = g(x, y)}$ whose values are given in the following table.**

Values of $\mathbf{z = g(x, y)}$

	$\mathbf{x = -3}$	$\mathbf{x = 0}$	$\mathbf{x = 3}$
$\mathbf{y = 2}$	**8**	**14**	**20**
$\mathbf{y = 0}$	**14**	**20**	**26**
$\mathbf{y = -2}$	**20**	**26**	**32**

Answer: $g(x, y) = 2x - 3y + 20$.

Example 4 **Find a formula for the linear function $\mathbf{z = h(x, y)}$ whose level curves are given in Figure 3.**

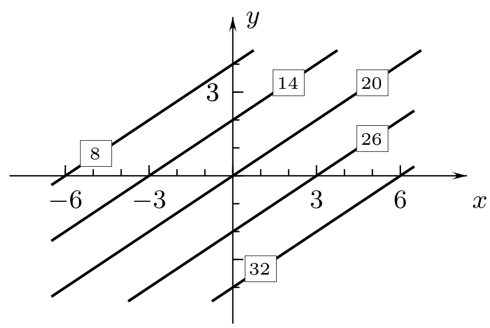


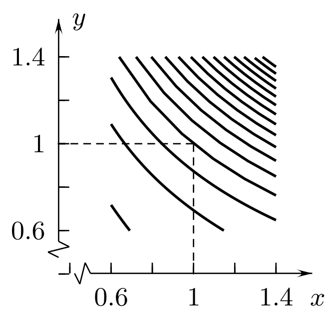
FIGURE 3

Answer: $h(x, y) = 2x - 3y + 20$. (Notice that h is the same as the function g from Example 3.)

Zooming in on level curves of a nonlinear $\mathbf{z = f(x, y)}$

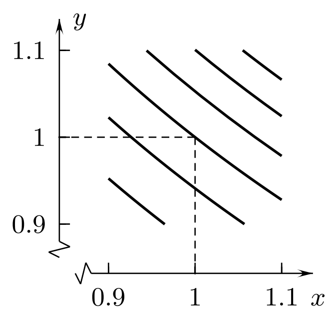
If a function $y = f(x)$ of one variable has a derivative at x_0 and the graph $y = f(x)$ is generated by a calculator or computer in a small enough window containing the point $(x_0, f(x_0))$, the displayed portion of the graph will look like a line. This occurs because the graph is closely approximated by the tangent line near that point.

Graphs of functions of two variables with continuous first derivatives are closely approximated by planes in small windows. Consequently, their level curves at equal z-increments look like equally spaced parallel lines in small windows. This is illustrated by the level curves of $K(x, y) = 3x^2y^3 + x$ in Figures 4 through 6. The level curves look more like equally spaced parallel lines in Figure 5 than Figure 4, and even more like equally spaced parallel lines in Figure 6. These approximate closely the level lines of the function whose graph is the tangent plane to the graph of the function $z = K(x, y)$ at $x = 1, y = 1$.



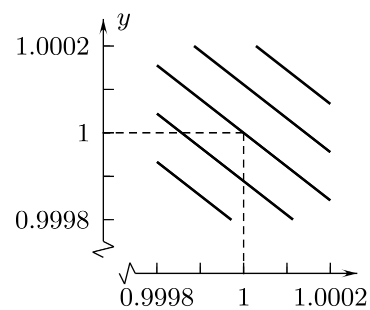
$K = 1, 2, 3, \ldots, 17$

FIGURE 4



$K = 3, 3.5, 4, \ldots, 5.5$

FIGURE 5



$K = 3.998, 3.999, \ldots, 4.002$

FIGURE 6

Example 5 **Give an equation of the tangent plane to the graph of $\mathbf{f(x, y) = x^3y^4}$ at $\mathbf{x = 1, y = 2}$.**

Answer: Tangent plane: $z = 16 + 48(x - 1) + 32(y - 2)$

Example 6 **The radius of a right circular cylinder is measured to be $\mathbf{r = 10 \pm 0.01}$ centimeters (meaning that it is measured to be 10 centimeters with an error $\mathbf{\le 0.01}$) and its height is measured to be $\mathbf{h = 15 \pm 0.005}$ centimeters (meaning that it is measured to be 15 centimeters with an error $\mathbf{\le 0.005}$). Use a tangent-plane approximation to estimate the maximum possible error in the volume of the cylinder that is calculated using $\mathbf{r = 10}$ and $\mathbf{h = 15}$.**

Answer: The maximum possible error is approximately $3.5\pi \doteq 11$ cubic centimeters.

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:[‡]

Section 14.6: Examples 1 through 3

[‡]The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.