(10/1/08)

Math 20C. Lecture Examples.

Section 14.6. The Chain Rule[†]

Example 1 **Find the t-derivative of $\mathbf{z = f(x(t), y(t))}$ at $\mathbf{t = 0}$ where $\mathbf{f(x, y) = x^2 + y^2}$, $\mathbf{x(0) = 5, y(0) = -1, x'(0) = 3}$, and $\mathbf{y'(0) = -4}$.**

Answer: $\dfrac{d}{dt}[f(x, y)]\Big|_{t=0} = 19$

Example 2 **What is the t-derivative of $\mathbf{z = f\,(x(t), y(t))}$ at $\mathbf{t = 1}$ if $\mathbf{x(1) = 2, y(1) = 3}$, $\mathbf{x'(1) = -4}$, $\mathbf{y'(1) = 5}$, $\mathbf{f_x(2, 3) = -6}$, and $\mathbf{f_y(2, 3) = 7}$?**

Answer: $\dfrac{d}{dt}\left[f\left(x(t), y(t)\right)\right]\Big|_{t=1} = 59$

Example 3 **A plane uses gasoline at the rate of $\mathbf{r = r(h, v)}$ gallons per hour when it is flying at an altitude of h feet and its air speed is v knots (nautical miles per hour). At one moment it has an altitude of 8000 feet, its speed is 120 knots, its height is increasing 500 feet per minute and it is accelerating 3 knots per minute. At what rate is its rate of gasoline consumption increasing or decreasing at that moment if $\mathbf{r_h(8000, 120) = -2 \times 10^{-4}}$ gallons per hour per foot and $\mathbf{r_v(8000, 120) = 0.13}$ gallons per hour per knot?**[(1)]

Answer: The plane's rate of fuel consumption is increasing 0.29 gallons per hour per minute.

Example 4 **What is the s-derivative of $\mathbf{z = f(st^2, te^s)}$ at $\mathbf{s = 0, t = 2}$ if the first-order derivatives of $\mathbf{z = f(x, y)}$ have the values $\mathbf{f_x(0, 2) = 10}$ and $\mathbf{f_y(0, 2) = -5}$?**

Answer: $\dfrac{\partial}{\partial s}[f(st^2, te^s)]\Big|_{s=0, t=2} = 30$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/~ashenk/:[‡]

Section 14.4: Examples 1–6

[†] Lecture notes to accompany Section 14.6 of *Calculus, Early Transcendentals* by Rogawski.

[(1)] Data adapted from *Cessna 172N Information Manual* , Wichita Kansas: Cessna Aircraft Company, 1978, p.5-16.

[‡] The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.