Math 20C. Lecture Examples.

Section 14.7, Part 1. Maxima and minima: The first-derivative test†

Example 1  The function \( g(x, y) = 2x^2 + y^2 - 2xy - 2y \) has a global minimum. Find it.

Answer: The global minimum is \(-2\).

Example 2  Find, without using calculus, the global maximum of

\[
 f(x, y, z) = \frac{1}{(x - y)^2 + 1} + e^{-z^2}
\]

and where it occurs.

Answer: The global maximum is 2 at the points \((x, x, 0)\) for all \(x\).

Example 3  The function \( M(x, y) = \frac{-5y}{x^2 + y^2 + 1} \), whose graph is shown in Figure 1, has a global maximum and a global minimum. What are they and where do they occur?

![Figure 1](image)

Answer: The global maximum is \(\frac{5}{2}\) at \((0, -1)\) and the global minimum is \(-\frac{5}{2}\) at \((0, 1)\).

Example 3  Suppose that rectangular boxes with no tops (Figure 2) are to be manufactured so that each has a volume of 6 cubic feet. The boxes are to be made from material that costs 6 dollars per square foot for the bottoms, 2 dollars per square foot for the fronts and backs, and 1 dollar per square foot for the sides. What dimensions would minimize the cost of manufacturing each box?

![Figure 2](image)

Answer: The boxes should be manufactured to be 1 foot wide, 2 feet long, and 3 feet high.

†Lecture notes to accompany Section 14.7, Part 1 of Calculus, Early Transcendentals by Rogawski.
**Example 5**  The function \( N(x, y) = \frac{1}{x} + \frac{1}{y} + xy \) has a minimum for \( x > 0, y > 0 \). What is it and where does it occur?

**Answer:** The minimum is 3 at (1, 1).

**Example 6**  What are the values of \( x \) and \( y \) such that the sum of the squares of the distances from the point \((x, y)\) to the four points \((a_1, b_1), (a_2, b_2), (a_3, b_3)\), and \((a_4, b_4)\) is a minimum?

**Answer:** \( x = \frac{1}{4}(a_1 + a_2 + a_3 + a_4) \bullet y = \frac{1}{4}(b_1 + b_2 + b_3 + b_4) \) (The averages)

**Interactive Examples**

Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/:

Section 15.1: Examples 1–6

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(Note: The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.)