Math 20C. Lecture Examples.

Section 14.7, Part 1. Maxima and minima: The first-derivative test†

Example 1 **The function $g(x,y) = 2x^2 + y^2 - 2xy - 2y$ has a global minimum. Find it.**

Answer: The global minimum is -2.

Example 2 **Find, without using calculus, the global maximum of**

$$f(x,y,z) = \frac{1}{(x-y)^2+1} + e^{-z^2}$$ **and where it occurs.**

Answer: The global maximum is 2 at the points $(x, x, 0)$ for all x.

Example 3 **The function $M(x,y) = \dfrac{-5y}{x^2+y^2+1}$, whose graph is shown in Figure 1, has a global maximum and a global minimum. What are they and where do they occur?**

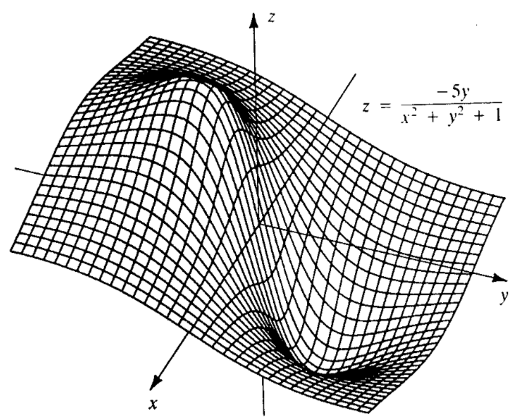

FIGURE 1

Answer: The global maximum is $\frac{5}{2}$ at $(0,-1)$ and the global minimum is $-\frac{5}{2}$ at $(0,1)$.

Example 3 **Suppose that rectangular boxes with no tops (Figure 2) are to be manufactured so that each has a volume of 6 cubic feet. The boxes are to be made from material that costs 6 dollars per square foot for the bottoms, 2 dollars per square foot for the fronts and backs, and 1 dollar per square foot for the sides. What dimensions would minimize the cost of manufacturing each box?**

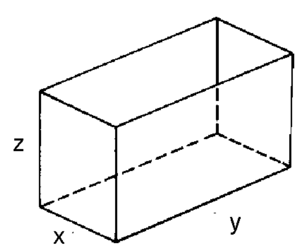

FIGURE 2

Answer: The boxes should be manufactured to be 1 foot wide, 2 feet long, and 3 feet high.

†Lecture notes to accompany Section 14.7, Part 1 of *Calculus, Early Transcendentals* by Rogawski.

Example 5 **The function $\mathbf{N(x,y) = \dfrac{1}{x} + \dfrac{1}{y} + xy}$ has a minimum for $\mathbf{x > 0, y > 0}$. What is it and where does it occur?**

Answer: The minimum is 3 at $(1, 1)$.

Example 6 **What are the values of $\mathbf{x}$ and $\mathbf{y}$ such that the sum of the squares of the distances from the point $\mathbf{(x, y)}$ to the four points $\mathbf{(a_1, b_1), (a_2, b_2), (a_3, b_3)}$, and $\mathbf{(a_4, b_4)}$ is a minimum?**

Answer: $x = \frac{1}{4}(a_1 + a_2 + a_3 + a_4)$ • $y = \frac{1}{4}(b_1 + b_2 + b_3 + b_4)$ (The averages)

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/˜ashenk/:‡

Section 15.1: Examples 1–6

‡The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.