Math 20C. Lecture Examples.

Sections 15.1 and 15.2. Double integrals†

Example 1  What is the value of \( \iint_{R} \sqrt{9 - x^2 - y^2} \, dx \, dy \) where \( R \) is the disk \( x^2 + y^2 \leq 9 \)?

Answer: \( \iint_{R} \sqrt{9 - x^2 - y^2} \, dx \, dy = \text{[Volume of the hemisphere in Figure A1]} = 18\pi \)

Figure A1

Example 2  Evaluate \( \iint_{R} 2xy \, dx \, dy \), where \( R \) is the region bounded by the curve \( y = x^2 \) and the lines \( y = 0 \) and \( x = 2 \) by performing a \( y \)-integration first.

Answer:  \( \iint_{R} 2xy \, dx \, dy = \frac{32}{3} \)

Figure A2

Example 3  Evaluate the integral of Example 2 by performing an \( x \)-integration first.

Answer:  \( \iint_{R} 2xy \, dx \, dy = \frac{32}{3} \)

Figure A3

†Lecture notes to accompany Sections 15.1 and 15.2 of Calculus, Early Transcendentals by Rogawski.
Example 4  
Evaluate \( \int_0^8 \int_{x^{1/3}}^2 \sin(y^4) \, dy \, dx \) by reversing the order of integration.

Answer: Figures A4a and A4b  
\[
\int_0^8 \int_{x^{1/3}}^2 \sin(y^4) \, dy \, dx = \int_0^2 \int_0^{x^{1/3}} \sin(y^4) \, dx \, dy = \frac{1}{4} - \frac{1}{4} \cos(16)
\]

![Figure A4a](image1.png)  
![Figure A4b](image2.png)

Example 5  
What is the volume of the solid \( V \) bounded by the surface \( z = \sin x \sin y \) and the plane \( z = -2 \) for \( 0 \leq x \leq \pi, 0 \leq y \leq \pi \) ?

Answer: The projection of the solid on the \( xy \)-plane is the square in Figure A5.  
\[
[\text{Volume}] = 4 + 2\pi^2
\]

![Figure A5](image3.png)

Example 6  
A plate that occupies the region \( R : 0 \leq x \leq 2, 0 \leq y \leq x^2 \) with distances measured in feet has density \( \rho(x, y) = 3xy^2 \) pounds per square foot at \((x, y)\). How much does the plate weigh?

Answer: Figure A6  
\[
[\text{Weight}] = \int \int_R 3xy^2 \, dx \, dy = 32 \text{ pounds}
\]

![Figure A6](image4.png)
**Example 7**  Figure 1 shows level curves of the population density $z = p(x, y)$ (people per square mile) in a city. Find the approximate population in the region $R$: $0 \leq x \leq 4$, $0 \leq y \leq 4$. Use a Riemann sum corresponding to a partition into four squares.

![Level curves of $z = p(x, y)$](FIGURE 1)

**Answer:** Population $= \int \int_{R} p(x, y) \, dy \, dx$  
• One approach: Approximate the integral with the Riemann sum corresponding to the partition of $R$ into four equal square subregions of Figure A7 with the population density evaluated at their midpoints.  
• The population is approximately 58,000.

![Figure A7](miles)

**Example 8**  What is the average value of $f(x, y) = ye^{y^2}$ on the region $R: 0 \leq y \leq \sqrt{x}$, $0 \leq x \leq 1$?

**Answer:**  
• [Area of $R$] $= \frac{2}{3}$  
• $\int \int_{R} ye^{y^2} \, dx \, dy = \frac{1}{2}(e - 2)$  
• [Average value] $= \frac{1}{4}(e - 2)$

![Figure A8](miles)

**Interactive Examples**

Work the following Interactive Examples on Shenk’s web page, [http://www.math.ucsd.edu/~ashenk/](http://www.math.ucsd.edu/~ashenk/):†

- Section 16.1: Examples 1 through 5
- Section 16.2: Examples 1, 2a, and 3

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†The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.