Math 20C. Lecture Examples.

Section 15.3. Triple integrals†

Example 1 What is the geometric significance of $\iiint_V 1\,dx\,dy\,dz$?

Answer: $\iiint_V 1\,dx\,dy\,dz = [\text{Volume of } V]$

Example 2 Express $\iiint_V 4xyz\,dx\,dy\,dz$ as an iterated integral of the form,

$$\iint_R \left\{\int_{z=g(x,y)}^{z=h(x,y)} f(x,y,z)\,dz\right\} dx\,dy$$

where V is the box defined by $0 \le x \le 3$, $0 \le y \le 2$, $0 \le z \le 1$.

Answer: $\iiint_V 4xyz\,dx\,dy\,dz = \iint_R \int_{z=0}^{z=1} 4xyz\,dz\,dy\,dx$ with R: $0 \le x \le 3, 0 \le y \le 2$ in Figure A2.

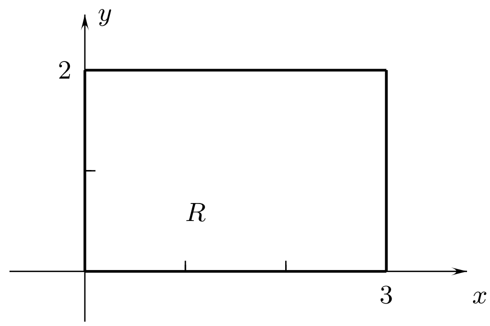

Figure A2

Example 3 Evaluate the integral $\iiint_V 4xyz\,dx\,dy\,dz = \iint_R \int_{z=0}^{z=1} 4xyz\,dz\,dy\,dx$ from Example 2, where R is the rectangle in Figure A2.

Answer: Figure A3 • $\iiint_V 4xyz\,dx\,dy\,dz = 18$

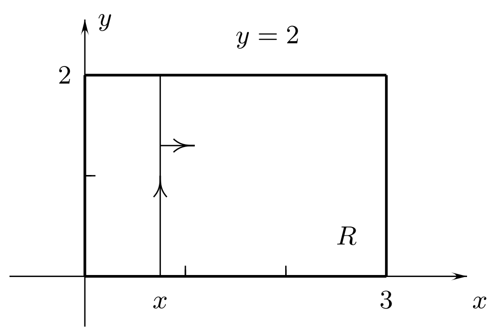

Figure A3

†Lecture notes to accompany Section 15.3 of *Calculus, Early Transcendentals* by Rogawski.

Example 4 The solid V in xyz-space with distances measured in meters is bounded by $z = 0$, $z = y, y = x^2$, and $y = 1$. Its density at (x, y, z) is $\rho(x, y, z) = 8yz$ kilograms per cubic meter. **(a)** Express the mass of V as an iterated integral. **(b)** Evaluate the integral.

Answer: (a) Figure A4 • $[\text{Mass}] = \iiint_V \rho(x, y, z)\, dx\, dy\, dz = \iint_R \int_{z=0}^{z=y} 8yz\, dx\, dy\, dz$
$= \int_{z=-1}^{x=1} \int_{y=x^2}^{y=1} \int_{z=0}^{z=y} 8yz\, dz\, dy\, dx$ **(b)** $[\text{Total charge}] = \frac{16}{9}$ kilograms

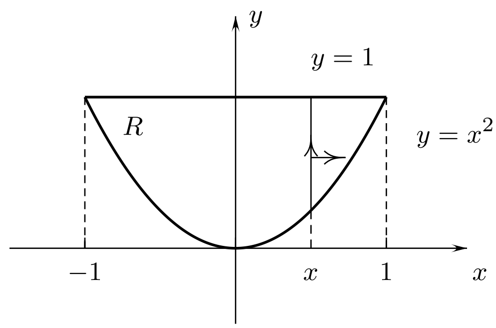

Figure A4

Example 5 What is the average value of $g(x, y, z) = xe^y \sin z$ on the cube V: $0 \le x \le 2, 0 \le y \le 2$, $0 \le z \le 2$?

Answer: $[\text{Average value}] = \frac{1}{4}(e^2 - 1)[1 - \cos(2)]$