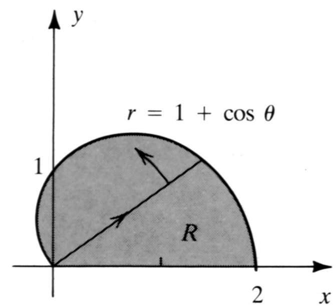
(9/7/08)

Math 20C. Lecture Examples.

Section 15.4. Integrals in polar and cylindrical coordinates†

Example 1 **(a)** Express $\iint_R y\,dx\,dy$ as an iterated integral in polar coordinates, where R is the region bounded by the x-axis and half cardioid $r = 1 + \cos\theta, 0 \le \theta \le \pi$ in Figure 1.
(b) Evaluate the integral.

FIGURE 1



Answer: (a) $\iint_R y\,dx\,dy = \int_{\theta=0}^{\theta=\pi}\int_{r=0}^{r=1+\cos\theta} r^2 \sin\theta\,dr\,d\theta$ **(b)** $\iint_R y\,dx\,dy = \frac{4}{3}$

Example 2 Find the average value of $f(x,y) = \sqrt{x^2+y^2+1}$ on the circle R: $x^2 + y^2 \le 1$.

Answer: Figure A2a ● [Average value] $= \frac{2}{3}(2^{3/2} - 1)$

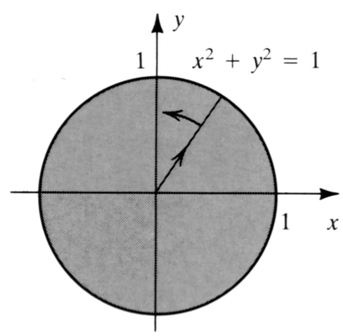


Figure A2

†Lecture notes to accompany Section 15.4 of *Calculus, Early Transcendentals* by Rogawski.

Example 3 Evaluate $\iint_R xy\,dx\,dy$, where R is bounded by the semicircle $y = \sqrt{x - x^2}$ and the x-axis.

Answer: $y = \sqrt{x - x^2}$ has the polar equation $r = \cos\theta$, $0 \le \theta \le \frac{1}{2}\pi$ • Figure A3 •
$\iint_R xy\,dx\,dy = \frac{1}{24}$

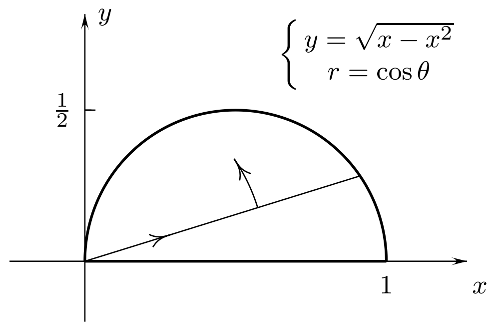


Figure A3

Example 4 **(a)** Express $\iiint_V 2z\,dx\,dy\,dz$ as an iterated integral in cylindrical coordinates, where V is the hemisphere of radius 1 in Figure 2. **(b)** Evaluate the integral.

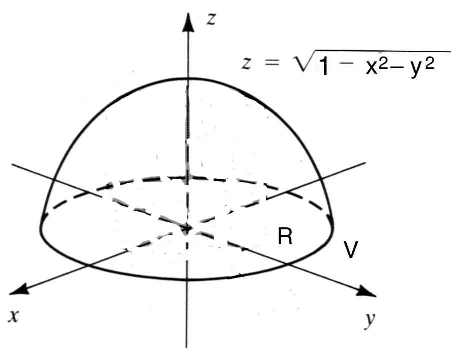


FIGURE 2

Answer: (a) $\iiint_V 2z\,dx\,dy\,dz = \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=1}\int_{z=0}^{z=\sqrt{1-r^2}} 2rz\,dz\,dr\,d\theta$ •

(This integration procedure is shown in Figure A4.) **(b)** $\iiint_V 2z\,dx\,dy\,dz = \frac{1}{2}\pi$

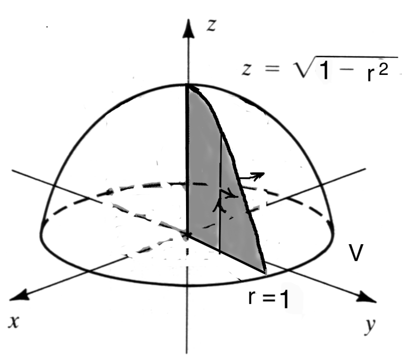


Figure A4