Math 20C (Shenk). Workshop Problems.

1. Which of the angles in the triangle with vertices $P = (1, -2, 0), Q = (2, 1, -2)$, and $R = (6, -1, -3)$ is a right angle?

Answer: The right angle is at $Q = (2, 1, -2)$.

2. Give parametric equations of the line through the point $P = (0, 4, 10)$ and perpendicular to the plane $4x - 5y + 6z = 2$.

Answer: L: $x = 4t,\ y = 4 - 5t,\ z = 10 + 6t$

3. Give an equation for the plane through the origin and parallel to the plane $3x - y + z = 1000$.

Answer: $3x - y + z = 0$

4. An airplane is at $x = 300t, y = -400t, z = 3$ (miles) at time t (hours) in xyz-space with the positive z-axis pointing up and the origin on the ground. Find its constant vector velocity and constant speed.

Answer: [Velocity] $= \mathbf{v}(t) = \langle 300, -400, 0\rangle$ miles per hour • [Speed] $= 500$ miles per hour

5 Figure 1 shows an object's path and the tangent and normal lines to the path at a point P where the radius of curvature is 8 feet. When the object is at P, it is moving $\sqrt{40}$ feet per second and is speeding up eight feet per second^2. Draw its approximate acceleration vector at that point, using the scales on the axes to measure the components.

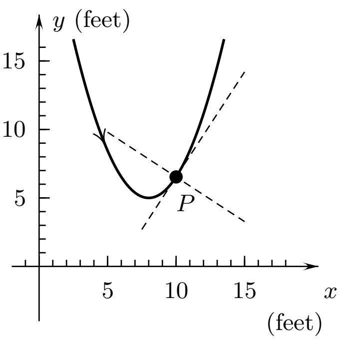

FIGURE 1

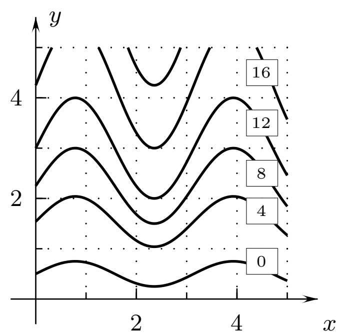

FIGURE 2

Answer: $\mathbf{a} = 8\mathbf{T} + 5\mathbf{N}$

6. Flnd the approximate maximum and minimum values of $W_y(x, y)$ for $0 \le x \le 5, 1 \le y \le 4$, where $z = W(x, y)$ is the the function whose level curves are in Figure 2.

Answer: [Maximum W_y] ≈ 8 • [Minimum W_y] ≈ 3

7. **(a)** Draw and label the level curves of $g(x, y) = \frac{1}{2}x^2 + \frac{1}{2}y^2$ through the points $(1, 1), (1, -2)$, and $(-3, -1)$.
(b) draw ∇g at those points, using the scales on the axes to measure its components.

Answer: Level curves: $\frac{1}{2}x^2 + \frac{1}{2}y^2 = 1$, $\frac{1}{2}x^2 + \frac{1}{2}y^2 = \frac{5}{2}$, $\frac{1}{2}x^2 + \frac{1}{2}y^2 = 5$
(b) $\nabla g(1, 1) = \langle 1, 1\rangle$ • $\nabla g(1, -2) = \langle 1, -2\rangle$ • $\nabla g(-3, -1) = \langle -3, -1\rangle$

8. Find the gradient of $f(x, y) = \ln(xy)$ at $(5, 10)$.

Answer: $\nabla f(5, 10) = \langle \frac{1}{5}, \frac{1}{10}\rangle$

9. Give an equation of the tangent plane to $z = \frac{1}{2}\ln(x^2 + y^2)$ at $x = 3, y = 4$.

Answer: Tangent plane: $z = \ln(5) + \frac{3}{25}(x - 3) + \frac{4}{25}(y - 4)$

10. Find the critical points of $f = 16x - x^2 + 8\ln y - y$ and use the Second Derivative Test to classify them.

Answer: Saddle point at $(8, 8)$

11. Figure 3 shows level curves of a function $z = f(x, y)$ and one level curve of a function $z = g_1(x, y)$. Figure 4 shows the same level curves of f and one level curve of another function $z = g_2(x, y)$. How do theses drawing relate to the method of Lagrange multipliers?

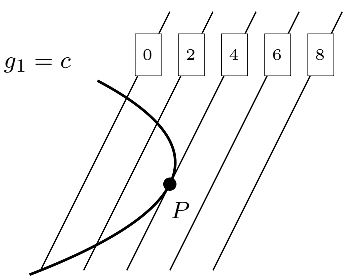

FIGURE 3

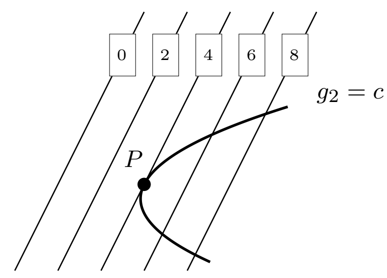

FIGURE 4

12. Evaluate $\displaystyle\iint_R 3x^2y^2\, dx\, dy$ whereis R bounded by $y = x, y = 2x$, and $x = 1$

Answer: $\displaystyle\iint_R 3x^2y^2\, dx\, dy = \frac{7}{6}$

13. What the average value of $g(x, y) = \sin x \sin y$ in the square R: $0 \le x \le \pi, 0 \le y \le \pi$?

Answer: [Average value] $= \dfrac{4}{\pi^2}$

14. Evaluate $\displaystyle\iint_R (x^2 + y^2)^{-2}\, dx\, dy$ with $R = \{(x, y) : 4 \le x^2 + y^2 \le 9\}$ by using polar coordinates.

Answer: $\displaystyle\iint_R (x^2 + y^2)^{-2}\, dx\, dy = \frac{5}{36}\pi$

15. What is the value of $\displaystyle\iiint_V 3z^2\, dx\, dy\, dz$ if V is bounded by $z = 0, z = x^2, x = 0, y = 0$, and $y = 1 - x$?

Answer: $\displaystyle\iiint_V 3z^2\, dx\, dy\, dz == \frac{1}{7} - \frac{1}{8} = \frac{1}{56}$