

Math 20C (Shenk). Summer, 2011. Exam 1 Solution.

1. $\mathbf{A} \cdot \mathbf{B} = \langle -2, 1, 3 \rangle \cdot \langle -1, 4, k \rangle = 2 + 4 + 3k = 6 + 3k \bullet \mathbf{A} \cdot \mathbf{B} = 0 \iff 6 + 3k = 0 \bullet k = -2$
2. $\mathbf{C} \cdot \mathbf{D} = \langle 1, 1, 1 \rangle \cdot \langle 1, 2, -2 \rangle = 1(1) + 1(2) + 1(-2) = 1 \bullet |\mathbf{C}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \bullet$
 $|\mathbf{D}| = \sqrt{1^2 + 2^2 + 2^2} = 3 \bullet \cos \theta = \frac{\mathbf{C} \cdot \mathbf{D}}{|\mathbf{C}| |\mathbf{D}|} = \frac{1}{3\sqrt{3}} \bullet \theta = \cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$
3. One solution: $\overrightarrow{OQ} = \langle 3, 3, 3 \rangle \bullet \overrightarrow{PS} = \langle -1 - 1, 4 - 1, 2 - 1 \rangle = \langle -2, 3, 1 \rangle$
 $\overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{QR} = \overrightarrow{OQ} + \overrightarrow{PS} = \langle 3, 3, 3 \rangle + \langle -2, 3, 1 \rangle = \langle 1, 6, 4 \rangle \bullet R = (1, 6, 4)$
4. One answer: Normal vector: $\mathbf{n} = \langle 1, -1, 2 \rangle \bullet$ Line: $\begin{cases} x = 3 + t \\ y = 1 - t \\ z = 2 + 2t \end{cases}$
5. One answer: $\overrightarrow{PQ} = \langle 2 - 1, 3 - 4, 0 - 2 \rangle = \langle 1, -1, -2 \rangle \bullet$ Line: $\begin{cases} x = 1 + t \\ y = 4 - t \\ z = 2 - 2t \end{cases}$
6. One solution: $\overrightarrow{OP} = \langle 2, 0, 3 \rangle \bullet \overrightarrow{OQ} = \langle 2, -1, 1 \rangle \bullet$
[Normal vector] = $\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 3 \\ 2 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 3 \\ -1 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & 0 \\ 2 & -1 \end{vmatrix} \mathbf{k}$
 $= [0(1) - 3(-1)]\mathbf{i} - [(2(1) - 3(2))\mathbf{j} + [2(-1) - 0(2)]\mathbf{k} = 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k} = \langle 3, 4, -2 \rangle \bullet$
Check: $\overrightarrow{PQ} \cdot \mathbf{n} = 2(3) + 0(4) + 3(-2) = 0 \bullet \overrightarrow{PR} \cdot \mathbf{n} = 2(3) + (-1)(-4) + 1(-2) = 0 \bullet$
Plane (using P): $3(x - 2) + 4y - 2(z - 3) = 0$
7. $\overrightarrow{OP} = \langle 1, 2, 3 \rangle \bullet \overrightarrow{OQ} = \langle 0, 1, 5 \rangle \bullet \overrightarrow{OR} = \langle 4, 0, 1 \rangle \bullet$
 $\overrightarrow{OP} \cdot \overrightarrow{OQ} \times \overrightarrow{OR} = \langle 1, 2, 3 \rangle \cdot [\langle 0, 1, 5 \rangle \times \langle 4, 0, 1 \rangle] = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 4 & 0 & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & 5 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 0 & 5 \\ 4 & 1 \end{vmatrix} + 3 \begin{vmatrix} 0 & 1 \\ 4 & 0 \end{vmatrix}$
 $= 1[(1)(1) - (5)(0)] - 2[(0)(1) - (5)(4)] + 3[(0)(0) - (1)(4)] = 1(1) - 2(-20) + 3(-4) = 1 + 40 - 12 = 29$
• [Volume] = $\frac{1}{6} \left| \langle 1, 2, 3 \rangle \cdot [\langle 0, 1, 5 \rangle \times \langle 4, 0, 1 \rangle] \right| = \frac{29}{6}$
8. Figure A8

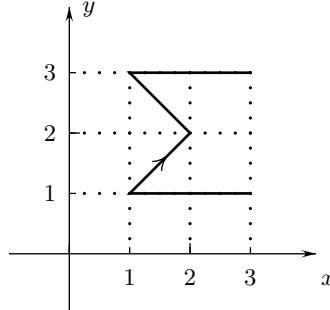


Figure A8

9. (a) $r(t) = \langle t - t^2, t^2 + t \rangle \bullet \mathbf{v}(t) = \frac{d}{dt} \langle t - t^2, t^2 + t \rangle = \langle 1 - 2t, 2t + 1 \rangle \bullet$

[Speed at time t] $= |\mathbf{v}(t)| = \sqrt{(1 - 2t)^2 + (2t + 1)^2} = \sqrt{1 - 4t + 4t^2 + 4t^2 + 4t + 1} = \sqrt{2 + 8t^2}$
 (b) The speed is a minimum at $t = 0$. $\bullet \mathbf{r}(0) = \langle 0, 0 \rangle \bullet \mathbf{v}(0) = \langle 1, 1 \rangle \bullet$ Figure A9

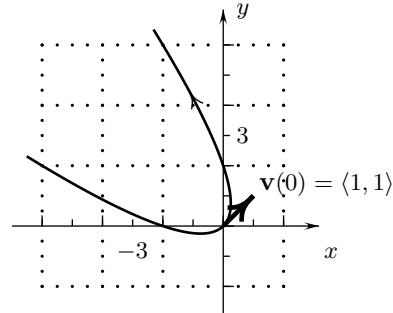


Figure A9