Math 20C (Shenk). Summer, 2010. Homework 1.

Section 12.1: 5, 9, 29, 39, 58, 59

Section 12.2: 5, 9, 19, 25, 27, 31, 39

Section 12.3: 1, 13, 19, 23, 41, 61, 71

Section 12.4: 5, 11, 13, 31, 34, 41, 43, 47

Section 12.5: 1, 11, 13, 15, 21, 23, 44

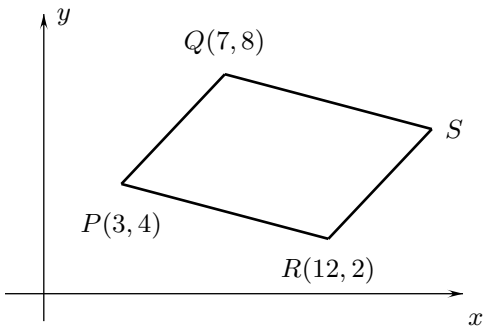
Section 11.1: 13, 20

Section 13.2: 3, 7, 13, 45, 55

Section 13.3: 1, 3

More problems to be turned in:

1. Three vertices of the parallelogram in the drawing below are $P = (3, 4), Q = (7, 8)$, and $R = (12, 2)$. Find the coordinates of the fourth vertex S without using vectors. Then find them by using vectors.



Answer: $S = (16, 6)$

2. A bucket of water is supported by two ropes fastened at the same point on the handle. The forces by the ropes on the bucket, relative to xyz-space with an upward pointing z-axis, are $\mathbf{F_1} = \langle 3, a, 6\rangle$ (pounds) and $\mathbf{F_2} = \langle b, -4, 5\rangle$ (pounds). What are the numbers a and b? How much does the bucket weigh?

Answer: $a = 4, b = -3$ and the bucket weighs 11 pounds.

3. A robot moving in an xy-plane with distances measured in meters is at $(120, 40)$ at $t = 0$ (minutes) and its velocity vector is $\mathbf{v}(t) = \langle -120\sin(2t),\ 80\cos(2t)\rangle$ (meters per minute) at time t . Find the robot's position vector $\mathbf{R} = \mathbf{R}(t)$ and describe its path.

Answer: $\mathbf{R}(t) = \langle 60\cos(2t) + 60,\ 40\sin(2t) + 40\rangle$ (meters) • The path is the ellipse in Figure A3 with center at $(60, 40)$, horizontal axis of length 120, and vertical axis of length 80.

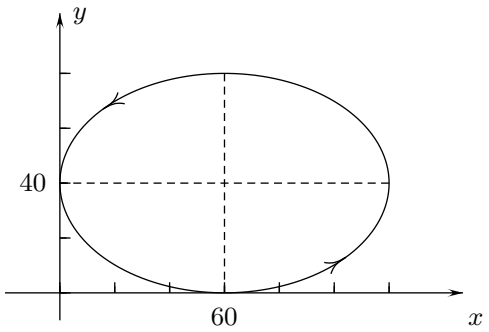
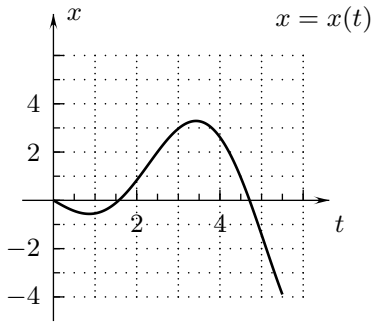


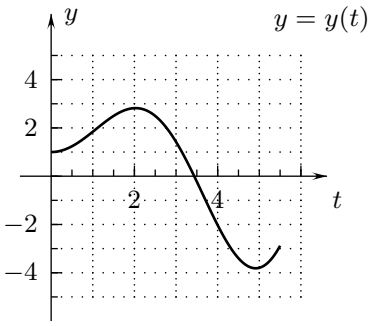
Figure A3

4. Figure 3 shows the curve C: $x = x(t), y = y(t), 0 \le t \le 5.5$, where $x = x(t)$ and $y = y(t)$ are the curves in Figures 1 and 2. Find the approximate velocity vector on C at $t = 3$ and draw it with the curve.



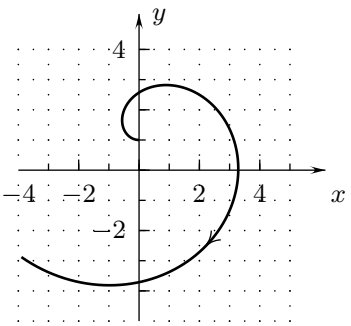
$x = x(t)$

FIGURE 1



$y = y(t)$

FIGURE 2



$C : x = x(t), y = y(t)$

FIGURE 3

Answer: Figures A4a and A4b • $\mathbf{v}(3) \approx \langle 1.5, -3 \rangle$ • Figure A4c

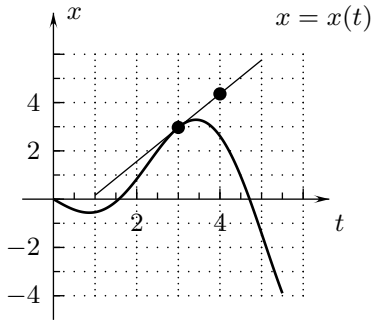


Figure A4a

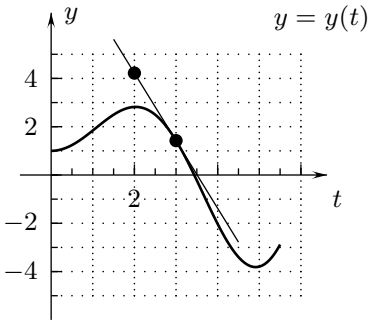


Figure A4b

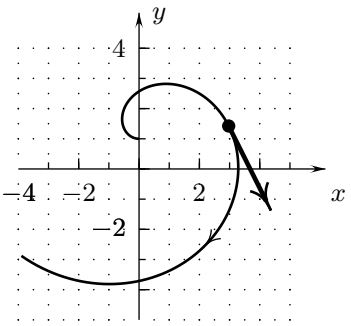


Figure A4c

5. An object is at $x = x(t), y = y(t)$ at time t for $0 \le t \le 3$, where $x = x(t)$ and $y = y(t)$ are the piecewise-linear functions whose graphs are given in Figures 4 and 5. **(a)** Find the velocity vectors at $t = 0.25, t = 1.25$, and $t = 2.25$. **(b)** Draw the object's path in an xy-plane with the velocity vectors from part (a). Use equal scales on the axes and the scale in Figure 6 to measure the components of the vectors.

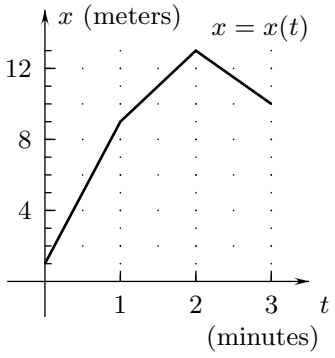


FIGURE 4

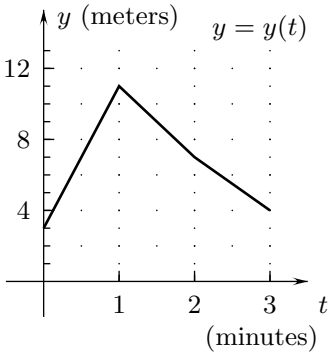


FIGURE 5

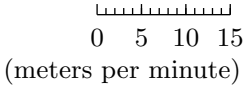
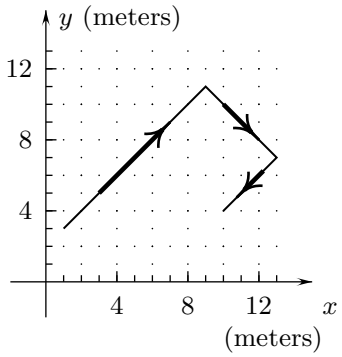


FIGURE 6

Answer: (a) $\mathbf{v}(0.25) = \langle 8, 8\rangle$ meters per minute ● $\mathbf{v}(1.25) = \langle 4, -4\rangle$ meters per minute ● $\mathbf{v}(2.25) = \langle -3, -3\rangle$ meters per minute ● Figure A5

Figure A5



6. The curve C: $x = t^2 - 2, y = 5t - t^3, -2.6 \le t \le 2, 6$ is shown in Figure 7. Find its velocity vector at $t = -1$ and draw it with the curve.

FIGURE 7