Math 20C (Shenk). Summer, 2010. Homework 3.

Section 14.2 (1, 19), Section 14.3 (9, 11, 19, 21, 35, 47, 51a, 59), Section 14.4 (11, 17, 31), Section 14.5 (5, 23, 31, 41), Section 14.7 (5, 7, 9)

1. Give equations of **(a)** the plane that has slope 3 in the positive x-direction and slope -5 in the positive y-direction, and whose z-intercept is 10 and **(b)** the plane that has slope -4 in the positive x-direction and slope 10 in the positive y-direction and contains the point $(5, 6, 7)$.

Answer: (a) $z = 3x - 5y + 10$ **(b)** $z = 7 - 4(x - 5) + 10(y - 6)$

2. Find the linear function $z = L(x, y)$ with the level curves in Figure 1.

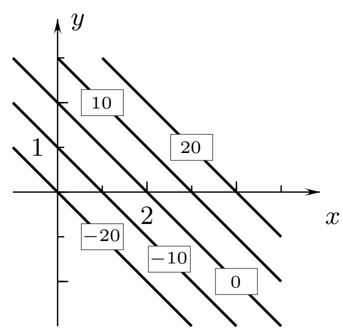

FIGURE 1

Answer: $L(x, y) = -20 + 10x + 10y$

3. Values of a function $z = P(x, y)$ are given in the next table. Is it possible that P is linear? If so, give a formula for such a function.

	$x = 0$	$x = 10$	$x = 20$
$y = 4$	-12	-16	-20
$y = 0$	0	-5	-8

Answer: The function cannot be linear.

4. The density of a saline solution of volume V (liters) containing w (grams) of salt is $\rho(V, w) = \dfrac{w}{V}$ grams per liter. **(a)** Give an equation of the tangent plane to $\rho = \rho(V, w)$ at $V = 10, w = 50$. **(b)** The volume is measured to be 10 liters with an error ≤ 0.01 liters and the mass of the salt is measured to be 50 grams with an error ≤ 0.06 grams. Use the tangent plane from part (a) to estimate the maximum possible error in the calculated volume.

Answer: (a) $\rho = 5 - \frac{1}{2}(V - 10) + \frac{1}{10}(w - 50)$ **(b)** [Maximum error] $\approx \frac{1}{2}(0.01) + \frac{1}{10}(0.06)$ $= 0.11$ grams per liter

5. What is the maximum directional derivative of $z = x^5 + y^3$ at $(1, 2)$? Give the unit vector in the direction of the maximum derivative.

Answer: [Maximum directional derivative] = 13 •
[Unit vector in the direction of the maximum derivative] = $\langle \frac{5}{13}, \frac{12}{13} \rangle$

6. Give unit vectors in the directions in which the directional derivative of $f(x, y) = x + \sin(5y)$ at $(2, 0)$ are zero.

Answer: $\mathbf{u} = \pm \dfrac{\langle 5, -1 \rangle}{\sqrt{26}}$

7. What are the critical points of $f(x, y) = x^4 + 32x - y^3$?

Answer: Critical point: $(-2, 0)$

8. Find the critical points of $f(x, y) = 3xy^2 + x^3 - 3x$.

Answer: Critical points: $(0, 1), (0, -1), (1, 0), (-1, 0)$

9. $G(x, y) = 2x^2 - x^4 + 10y - y^2$ has a global maximum. What is it and where does it occur?

Answer: [Global maximum] = 26 at $(\pm 1, 5)$

10. A three-dimensional rectangular frame is to be constructed with wire that costs \$1.00 per meter for the four horizontal edges parallel to the front, \$0.75 per meter for the four horizontal edges parallel to the sides, and \$1.50 per meter for the four vertical edges. The volume of the box enclosed by the frame is to be 24 cubic meters What dimensions minimize the cost?

Answer: With x the lengths of the four edges parallel to the front, y the lengths of the four edges parallel to the sides, and z the height, [Cost] $= C(x, y) = 4x + 3y + \dfrac{144}{xy}$ • The frame should be 3 meters wide, 4 meters deep and 2 meters high.

11. Find the point on the plane $z = 2x + 3y$ that is closest to the point $(4, 2, 0)$. (Minimize the square of the distance from $(4, 2, 0)$ to a point on the plane.)

Answer: The closest point is $(2, -1, 1)$

12. Find the critical points of $f = x^3 - y^3 - 3xy$ and use the Second-Derivative Test to classify them. The ıd its level curves are in Figure 3.

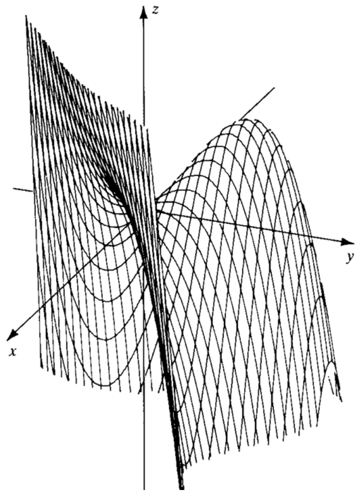

FIGURE 2

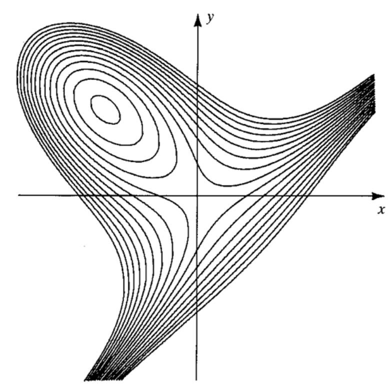

FIGURE 3

Answer: Saddle point at $(0, 0)$ • Local maximum at $(-1, 1)$

13. Show that the Second-Derivative Test fails at the critical point of $h(x, y) = \frac{1}{3}y^3 - x^2y$. Its graph is the "monkey saddle" in Figure 4. Its level curves are in Figure 5.

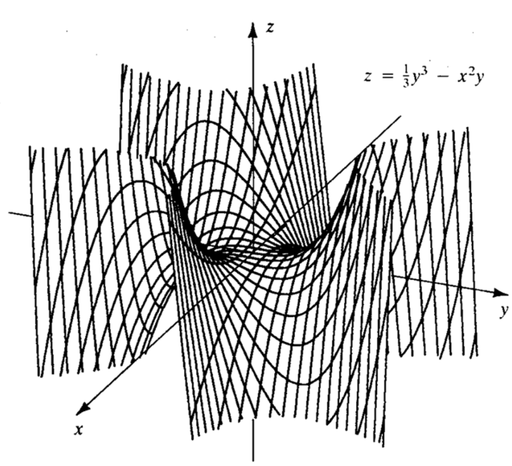

FIGURE 4

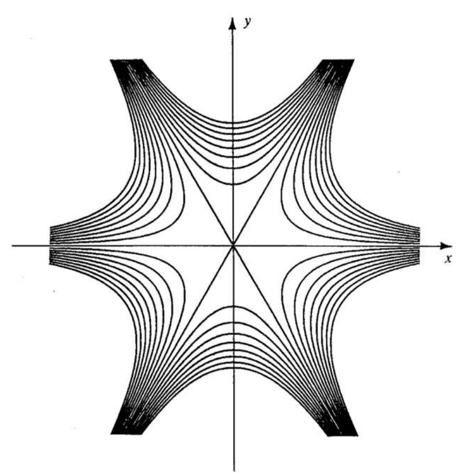

FIGURE 5