Math 20C (Shenk). Quiz 1 Solution. August 8, 2011.

1. $\mathbf{F} = \mathbf{F_1} + \mathbf{F_2} = \langle 7-11, -1+5\rangle = \langle -4, 4\rangle$ (pounds) • [Magnitude] $= |\mathbf{F}| = \sqrt{4^2+4^2} = 4\sqrt{2}$ (pounds) • [Angle of inclination] $= \frac{3}{4}\pi$ or $\frac{3}{4}\pi + 2n\pi$ with any integer n

2. $\overrightarrow{PQ} = \langle 2-1, 4-1, 3-2\rangle = \langle 1,3,1\rangle$ • $\overrightarrow{PR} = \langle 0-1, 1-1, 5-2\rangle = \langle -1,0,3\rangle$ •
$\overrightarrow{PQ}\cdot\overrightarrow{PR} = 1(-1)+3(0)+1(3) = 2$ •
$|\overrightarrow{PQ}| = \sqrt{1^2+3^2+1^2} = \sqrt{11}$ • $|\overrightarrow{PR}| = \sqrt{1^2+0^2+3^2} = \sqrt{10}$ • Let θ be the angle at P with $0 \le \theta \le \pi$.
• $\cos\theta = \dfrac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|} = \dfrac{2}{\sqrt{11}\sqrt{10}}$ • $\theta = \cos^{-1}\left(\dfrac{2}{\sqrt{11}\sqrt{10}}\right)$

3. $\overrightarrow{PQ} = \langle 5-1, 2-2, 1-3\rangle = \langle 4,0,-2\rangle$ is parallel to the line. • L: $\begin{cases} x = 1+4t \\ y = 2 \\ z = 3-2t \end{cases}$

4. **(a)** $\langle 1,2,3\rangle \times \langle 2,-1,0\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 2 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}\mathbf{k}$
$= [(2)(0)-(3)(-1)]\mathbf{i} - [(1)(0)-(3)(2)]\mathbf{j} + [(1)(-1)-(2)(2)]\mathbf{k}$
$= 3\mathbf{i} + 6\mathbf{j} - 5\mathbf{k} = \langle 3,6,-5\rangle$ **(b)** [Area] $= \frac{1}{2}|\langle 1,2,3\rangle \times \langle 2,-1,0\rangle| = \frac{1}{2}\sqrt{3^2+6^2+5^2} = \frac{1}{2}\sqrt{70}$

5. $\mathbf{n} =< 4,3,2 >$ is perpendicular to the plane. • Plane: $4(x-3)+3(y-2)+2(z-1) = 0$

6. Figure A5

$\begin{cases} x = x(t) \\ y = y(t) \\ 0 \le t \le 3 \end{cases}$

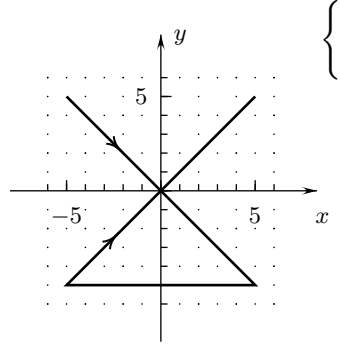
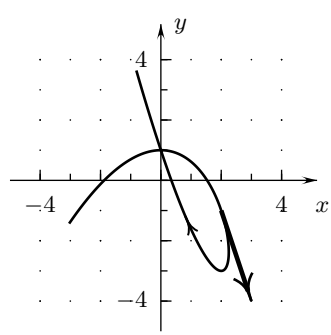


Figure A5

7. **(a)** $x' = 3-2t$ • $y' = 3t^2-6t$ • $x'(1) = 1$ • $y'(1) = -3$ • $\mathbf{v}(1) = \langle 1,-3\rangle$ •
$(x(1), y(1)) = (2,-1)$ • Draw an arrow from $(2,-1)$ to $(2+1, -1-3) = (3-4)$ • Figure A7
(b) [Speed] $= \sqrt{10}$ meters per second **(c)** [Length] $= \displaystyle\int_{-0.8}^{3.25} \sqrt{(3-2t)^2 + (3t^2-6t)^2}\, dt$



Figue A6