Math 20C (Shenk). Quiz 2. August 22, 2011.

Name _____ Section _____

Work alone and use no books, notes, or calculators. Put your work on 8.”‘ × 11” paper and staple it to the quiz when you turn it in.

1. What is the curvature at $x = 0$ of $y = e^{2x}$, oriented from left to right?

2. Use the formulas $\mathbf{v} = \dfrac{ds}{dt}\mathbf{T}$, $\mathbf{T} = \langle \cos\phi, \sin\phi \rangle$, $\mathbf{N} = \langle -\sin\phi, \cos\phi \rangle$, and $\kappa = \dfrac{d\phi}{ds}$ to derive the formula for the acceleration vector $\mathbf{a}$ in terms of $\dfrac{ds}{dt}, \dfrac{d^2s}{dt^2}, \mathbf{T}, \mathbf{N}$ and κ.

3. Figure 1 shows the path of an object in an xy-plane with distances measured in meters. The center of curvature of its path at the origin is the point $C = (4, 3)$. **(a)** (3 points) What is the curvature of its path at the origin? **(b)** (3 points) What is the unit normal vector $\mathbf{N}$ to its path at the origin?
(c) (4 points) When the object is at the origin, it is traveling 5 meters per minute and its speed is increasing 10 meters per minute^2. What is its acceleration vector at that time? Add it to the drawing, using the scales on the axes to measure its components.

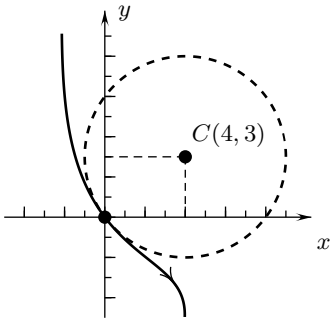

FIGURE 1

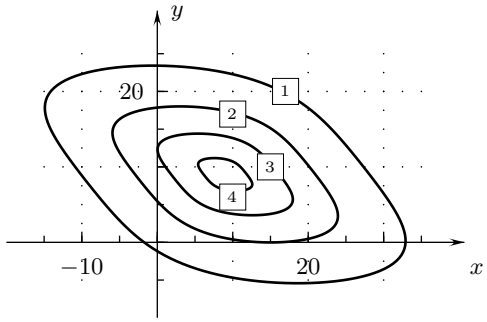

FIGURE 2

4. **(a)** Based on the level curves of $z = f(x, y)$ in Figure 2, what is the approximate value of $f_x(20, 10)$?
(b) What is the approximate directional derivative of f at $(20, 10)$ in the direction toward the origin?

5. Draw the graph of $G(x, y) = -1 - x^2 - y^2$.

6 Draw and label three level curves of the function $W(x, y) = 2y + x^2$.

7. Give a formula for the linear function $z = M(x, y)$ with the values $M(1, 1) = 10, M(6, 1) = 30$, and $M(1, 6) = 0$.

8. Find the derivative $\dfrac{\partial^2}{\partial x \partial y}[\sin(x^2 + y^2)]$.

9. Give an equation of the tangent plane to the graph of $f(x, y) = x^2y^3$ at $x = 5, y = -1$.

10. What is the directional derivative of $H(x, y) = xy$ at $(3, -1)$ in the direction toward $(7, 2)$?

11. What is the maximum directional derivative of $g(x, y, z) = \frac{1}{3}x^3 + 3\ln y + e^z$ at $(1, 3, 0)$?

Scores (10 points per problem)

1	2	3	4	5	6

7	8	9	10	11	Total