## Math 20C. Lecture Examples.

## Section 12.5. Planes in three space<sup> $\dagger$ </sup>

A vector  $\mathbf{n} = \langle a, b, c \rangle$  is said to be perpendicular or NORMAL to a plane in *xyz*-space if it is perpendicular to all lines in the plane (Figure 1).

**n** is perpendicularto all lines in the plane.FIGURE 1



(The point Q = (x, y, x) is in the plane if and only if  $\mathbf{n} \cdot \overrightarrow{PQ} = 0$ .

**Theorem 1** The plane through the point  $P = (x_0, y_0, z_0)$  and perpendicular to the nonzero vector  $\mathbf{n} = \langle a, b, c \rangle$  has the equation,

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$$

(Proof: The point Q = (x, y, x) is in the plane if and only if  $\mathbf{n} \cdot \overrightarrow{PQ} = 0$ .)

**Example 1** Give an equation of the plane through the point (2, 3, 4) and perpendicular to the vector  $\langle -6, 5, -4 \rangle$ .

**Answer:** -6(x-2) + 5(y-3) - 4(z-4) = 0

**Example 2** Give an equation for the plane through (6, 10, -3) and perpendicular to the line x = -3t, y = 6 + t, z = 4 - 7t.

**Answer:** 
$$-3(x-6) + (y-10) - 7(z+3) = 0$$

**Example 3** Give an equation of the plane through the point (1, -1, 2) that is parallel to the plane 3x - 5y + 6z = 10.

**Answer:** 3(x-1) - 5(y+1) + 6(z-2) = 0 or (written) 3x - 5y + 6z = 20.

**Example 4** Give an equation for the plane through the points P = (1,3,2), Q = (1,4,3), and R = (2,5,0).

**Answer:** -4(x-1) + (y-3) - (z-2) = 0, which simplifies to -4x + y - z = -3

**Example 5** Check the equation from Example 4 by verifying that the coordinates of P = (1, 3, 2), Q = (1, 4, 3), and R = (2, 5, 0) satisfy it. **Answer:** For P = (1, 3, 2): 4x - y + z = 4(1) - 3 + 2 = 3 • For Q = (1, 4, 3): 4x - y + z = 4(1) - 4 + 3 = 3 • For R = (2, 5, 0): 4x - y + z = 4(2) - 5 = 3

**Example 6** Give parametric equations of the line L through (3, -4, 5) and perpendicular to the plane 3x - 2y = 5.

**Answer:** L: x = 3 + 3t, y = -4 - 2t, z = 5

## Interactive Examples

Work the following Interactive Examples on Shenk's web page, http://www.math.ucsd.edu/~ashenk/:<sup>†</sup>

Section 12.5: Examples 3–5 and 6–9

<sup>&</sup>lt;sup>†</sup>Lecture notes to accompany Section 12.5 of Calculus, Early Transcendentals by Rogawski.

<sup>&</sup>lt;sup>†</sup>The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.