(7/25/08)

Math 20C. Lecture Examples.

Sections 11.1 and 13.1. Parametric equations and vector-valued functions[†]

Example 1 Draw the curve C: $x = x(t), y = y(t), 0 \le t \le 3$ in an xy-plane, where $x = x(t)$ and $y = y(t)$ are the piecewise-linear functions with graphs in Figures 1 and 2.

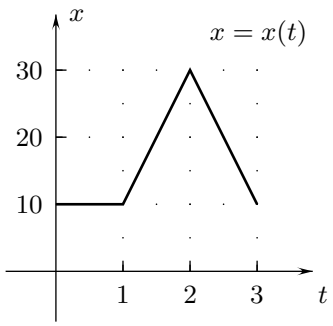


FIGURE 1

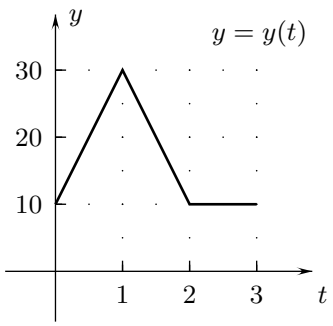


FIGURE 2

Answer: Use the values in the table below. ● Figure A1

t	0	1	2	3
$x = x(t)$	10	10	30	10
$y = y(t)$	10	30	10	10

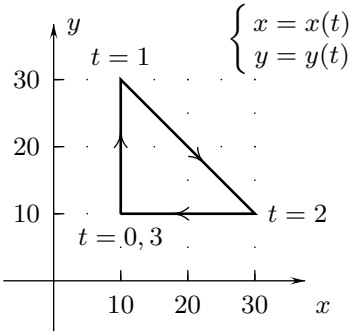


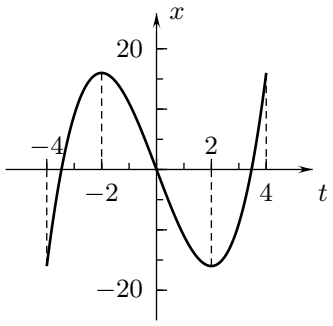
Figure A1

[†]Lecture notes to accompany Sections 11.1 and 13.1 of *Calculus, Early Transcendentals* by Rogawski.

Example 2 Sketch the curve C: $x = t^3 - 12t, y = 2t^2, -4 \le t \le 4$.

Answer: Draw $x = x(t)$ in a tx-plane and $y = y(t)$ in a ty-plane for $-4 \le t \le 4$ (Figures A2a and A2b). • Use these graphs to draw C: $x = t^3 - 12t, y = 2t^2, -4 \le t \le 4$ in an xy-plane. • Figure A2c

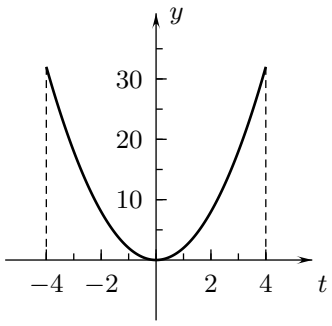
t	-4	-2	0	2	4
$x = t^3 - 12t$	-16	16	0	-16	16
$y = 2t^2$	32	8	0	8	32



$x(t) = t^3 - 12t$

$-4 \le t \le t$

Figure A2a



$y(t) = 2t^2$

$-4 \le t \le 4$

Figure A2b

$$\begin{cases} x = t^3 - 12t \\ y = 2t^2 \\ -4 \le t \le 4 \end{cases}$$

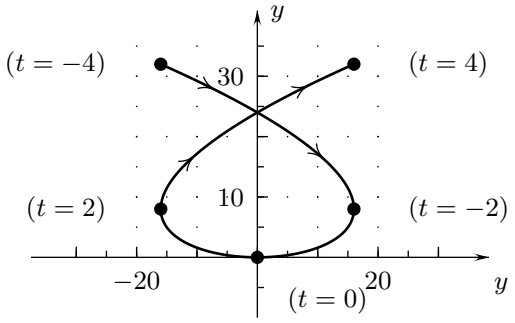


Figure A2c

Example 3 Draw the line with parametric equations $x = 2 + t, y = 1 + \frac{1}{2}t$.

Answer: Figure A3

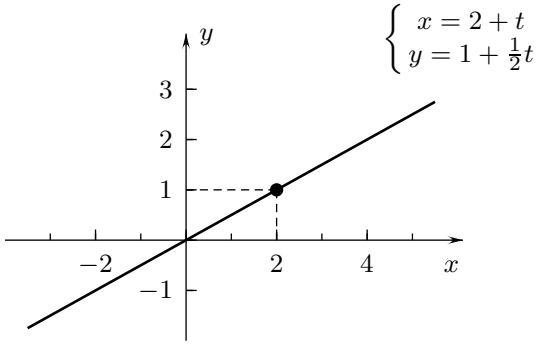


Figure A3

Parametric equations of ellipses

The ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with center at the origin, x-intercepts $x = \pm a$ and y-intercepts $y = \pm b$ in Figure 3 has the parametric equations,

$$x = a\cos t, y = b\sin t, 0 \le t \le 2\pi.$$

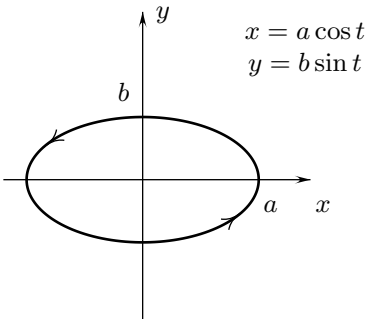


FIGURE 3

Example 4 Draw the ellipse C: $x = 2\cos t, y = 3\sin t, 0 \le t \le 2\pi$.

Answer: Figure A4

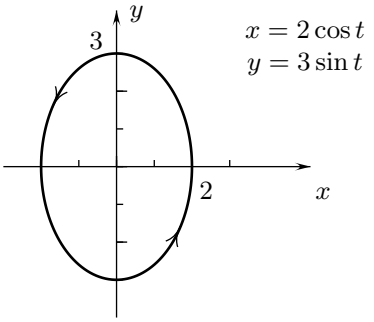


Figure A4

Example 5 When the circle of radius 1 in Figure 4 is rolled to the right along the x-axis, the point P that is initially at the origin generates the CYCLOID in Figure 5. Give parametric equations of this curve.

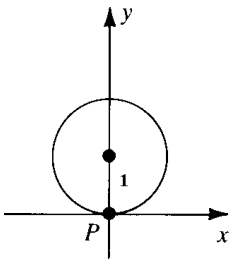


FIGURE 4

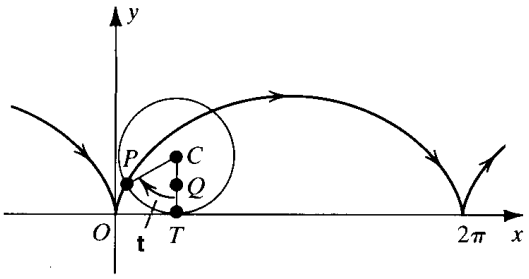


FIGURE 5

Answer: $x = t - \sin t, y = 1 - \cos t$

Theorem 1 *If a curve is given by $x = x(t), y = y(t)$ and $y'(a) \neq 0$, then the slope of the tangent line to the curve at $t = a$ is*

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}\Big|_{t=a} = \frac{y'(a)}{x'(a)}.$$

Example 6 What is the slope of the tangent line at $t = 3$ on the curve $x = t^3 - 12t, y = 2t^2$ in Figure 6?

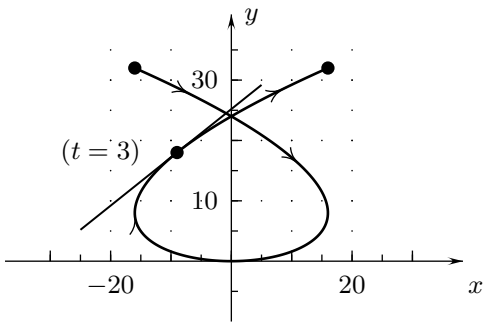


FIGURE 6

Answer: [Slope of the tangent line at $t = 3$] $= \frac{4}{5}$

Interactive Examples

Work the following Interactive Examples on Shenk's web page, http//www.math.ucsd.edu/~ashenk/:†

Section 13.1: Examples 1–5

†The chapter and section numbers on Shenk's web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.