(8/15/08)

Math 20C. Lecture Examples.

Section 14.1, Part 2. Functions of three or more variables†

Example 1 The frustum of a right circular cone with base of radius R (meters), top of radius r (meters), and height h (meters) (Figure 1) has volume $V(R,r,h) = \frac{1}{3}\pi(R^2+rR+r^2)h$. **(a)** What are the domain and range of V? **(b)** What does ~~$V(6,4,9)$~~ represent and what is its value if lengths are measured in meters? ~~(b)~~ **(c)** What is the geometric interpretation of $V(R,0,h)$?

$V(9,6,4)$

c

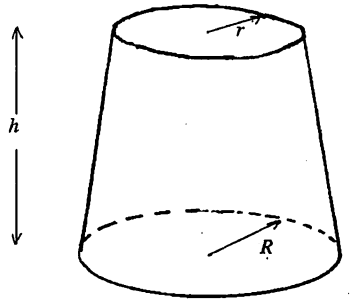


FIGURE 1

Answer: (a) The domain of V consists of all points (R,r,h) in three-dimensional Rrh-space such that $R \geq 0, r \geq 0, h \geq 0$. • Its range is the interval $[0,\infty)$ on a V-axis.
(b) $V(9,6,4)$ is the volume of a frustum of a right circular cone with base of radius 9 meters, top of radius 6 meters, and 4 meters high. • $V(9,6,4) = 228\pi$ cubic meters
(c) $V(R,0,h) = \frac{1}{3}\pi R^2 h$ is the volume of a right circular cone whose base has radius R and whose height is h. (Figure A1)

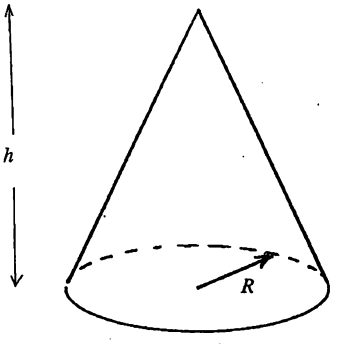


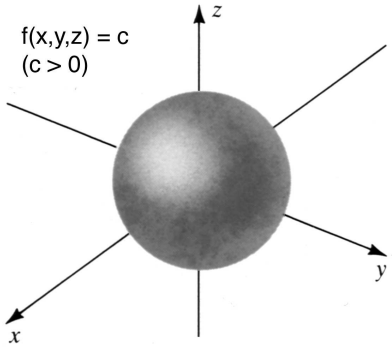
Figure A1

†Lecture notes to accompany Section 14.1, Part 2 of *Calculus, Early Transcendentals* by Rogawski.

Example 2 Describe the level surfaces of $f(x, y, z) = x^2 + y^2 + z^2$.

Answer: For $c = 0$, the level surface $f = c$ is the origin. • For $c > 0$ it is the surface of the sphere of radius $\sqrt{c}$ centered at the origin in Figure A2. • For $c < 0$ it is empty.

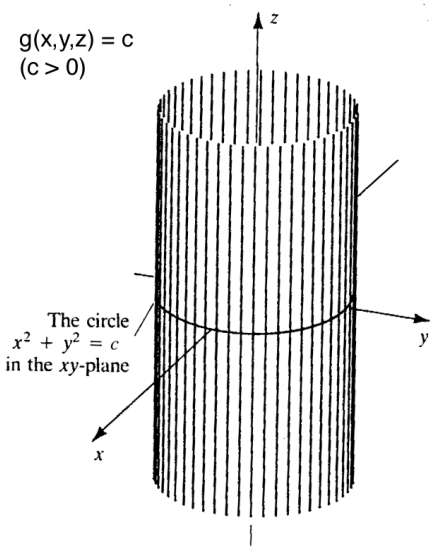
Figure A2



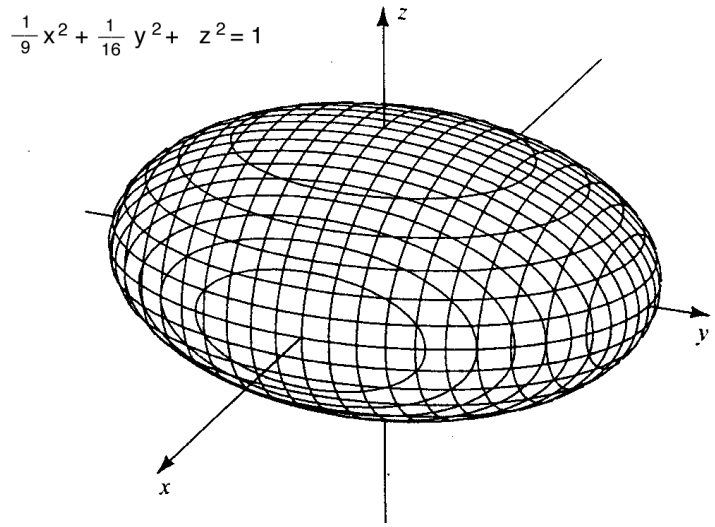
Example 3 Describe the level surfaces of $g(x, y, z) = x^2 + y^2$.

Answer: The level surface $g = c$ is the z-axis if $c = 0$, is the cylinder of radius $\sqrt{c}$ with the z-axis as axis in Figure A3 if $c > 0$, and is empty if $c < 0$.

Figure A3



Example 4 The level surface $h(x, y, z) = 1$ of $h(x, y, z) = \frac{1}{9}x^2 + \frac{1}{16}y^2 + z^2$ in Figure 2 is called an ELLIPSOID because all of its cross sections are ellipses. Find equations for and draw its cross sections **(a)** in the xz-plane, **(b)** in the yz-plane, and **(c)** in the xy-plane.



A level surface of
$h(x, y, z) = \frac{1}{9}x^2 + \frac{1}{16}y^2 + z^2$
FIGURE 2

Answer: (a) The cross section $\frac{1}{9}x^2 + z^2 = 1$ in the xz-plane is the ellipse with x-intercepts $x = \pm 3$ and z-intercepts $z = \pm 1$ in Figure A4a.
(b) The cross section $\frac{1}{16}y^2 + z^2 = 1$ in the yz-plane is the ellipse with y-intercepts $y = \pm 4$ and z-intercepts $z = \pm 1$ in Figure A4b.
(c) The the cross section $\frac{1}{9}x^2 + \frac{1}{16}y^2 = 1$ in the xy-plane is the ellipse with x-intercepts $x = \pm 3$ and y-intercepts $y = \pm 4$ in Figure A4c.

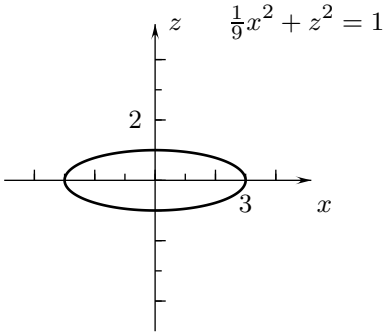


Figure A4a

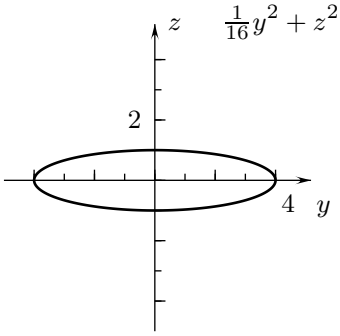


Figure A4b

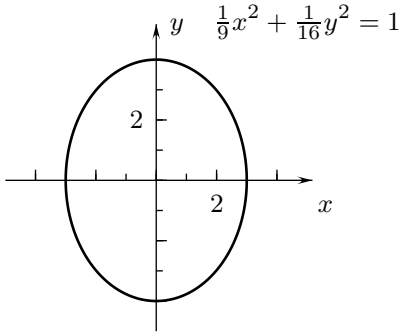


Figure A4c

Example 5 Describe the level surfaces $h = c$ of the function of Example 4 **(a)** with $c > 1$ and **(b)** with $0 < c < 1$.

Answer: (a) The level surfaces $h = c$ with $c > 1$ are geometrically similar ellipsoids outside the ellipsoid $h = 1$ of Figure 2. **(b)** The level surfaces $h = c$ with $0 < c < 1$ are geometrically similar ellipsoids inside the ellipsoid $h = 1$ of Figure 2.

Example 6 Figure 3 is a cross-sectional view of level surfaces of the Van Allen belts of cosmic radiation that surround the earth. (To visualize the level surfaces imagine that the curves as drawn are rotated around the north-south axis of the earth.) The radiation is measured in counts per second and the scale shows the distance in earth radii (≈ 4000 miles). At approximately what distances from the equator is the radiation the greatest?

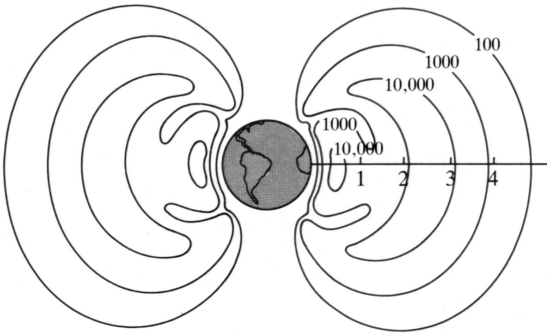


FIGURE 3

Answer: The radiation is greatest at approximately 0.5 earth radii and at approximately 2.5 earth radii from the equator.

Example 7 What is the domain of $P(w, x, y, z) = \sqrt{1 - w^2 - x^2 - y^2 - z^2}$?

Answer: The domain consists of all "points" (w, x, y, z) in $wxyz$-space such that $w^2 + x^2 + y^2 + z^2 \leq 1$.

Example 8 On July 21, 2006, Ford Motor stock sold for \$6.19 per share, Hewlett Packard stock sold for \$31,80 per share, Motorola stock sold for \$20.60 per share, Pepsi Cola stock sold for \$62.48 per share, and Yahoo stock sold for \$25.27 per share. Give a formula for the cost $C(x_1, x_2, x_3, x_4, x_5)$ on that day of x_1 shares of Ford Motor stock, x_2 shares of Hewlett Packard stock, x_3 shares of Motorola stock, x_4 shares of Pepsi Cola stock, and x_5 shares of Yahoo stock. What is the domain of this function?

Answer: $C(x_1, x_2, x_3, x_4, x_5) = 6.19x_1 + 31.80x_2 + 20.60x_3 + 62.48x_4 + 25.27x_5$ dollars. • The domain is the set of "points" $(x_1, x_2, x_3, x_4, x_5)$, such that x_1, x_2, x_3, x_4, and x_5 are all nonnegative integers