

Inverse functions and logarithms

OVERVIEW: Some applications require not only a function that converts a number x into a number y , but also its INVERSE, which converts y back into x . In this section we analyze inverse functions and discuss LOGARITHMS, which are the inverses of exponential functions.

Topics:

- **Inverse functions**
- **Changing variables**
- **Restricting domains**
- **Logarithms**
- **The common and natural logarithms**
- **Laws of logarithms**

Inverse functions

The function of Figure 1 converts the year t into the number $N = f(t)$ (millions) of cars that were registered in California at that time.⁽¹⁾ For example, starting with $t = 1980$ on the horizontal t -axis, moving vertically to the graph, and then horizontally to the vertical N -axis, we obtain $N = f(1980) = 17.4$. At the beginning of 1980 there were 17.4 million cars registered in California.

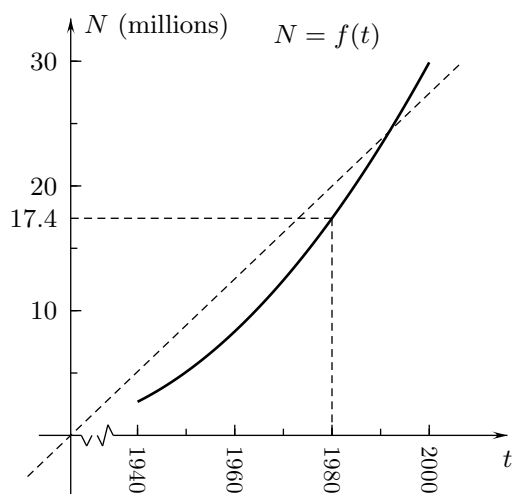


FIGURE 1

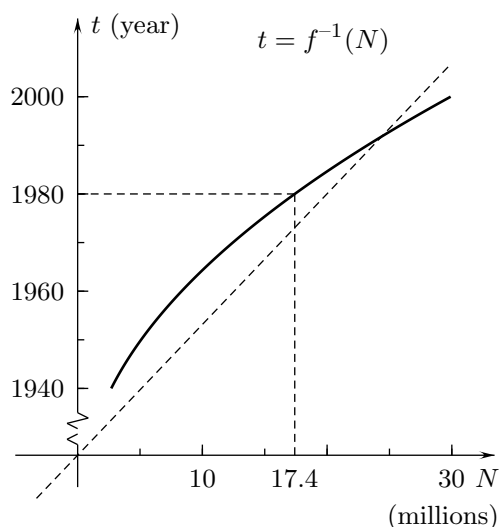


FIGURE 2

The INVERSE of f , denoted f^{-1} and read “ f inverse,” has the opposite effect.[†] It converts the number N into the time $t = f^{-1}(N)$ when there were N cars registered. We could use Figure 1 to study f^{-1} by having its variable (the independent variable) be on the vertical N -axis and its values on the horizontal t -axis. For example, we could start with $N = 17.4$ on the vertical axis, move horizontally to the graph, and then vertically to the horizontal t axis to obtain $t = 1980$. This gives $f^{-1}(17.4) = 1980$, which means that number of cars registered was 17.4 million at the beginning of 1980.

⁽¹⁾Data adapted from *CALPERG Citizen Agenda*, Vol. 18, No. 2, Los Angeles: CALPERG, 2000, p. 5.

[†]Because the symbol f^{-1} also denotes the reciprocal $1/f$ of f , its meaning must be determined from the context in which it is used.

This procedure for studying the inverse is, however, not convenient because we prefer to have the independent variable on the horizontal axis. To achieve this, we flip the drawing about the diagonal line that makes a 45° -degree angle with the positive x - and y -axes. This yields the graph $t = f^{-1}(N)$ of the inverse function in Figure 2, where the variable N is on the horizontal axis and the values t of the function are on the vertical axis. The graph of f^{-1} in Figure 2 is the mirror image of the graph of f in Figure 1 with respect to the diagonal line.

Here is a general definition:

Definition 1 Suppose that $y = f(x)$ is a function such that for each y in its range, the equation $y = f(x)$ has one and only one solution x . Then x is the value of the INVERSE FUNCTION $x = f^{-1}(y)$ at y . Thus,[†]

$$x = f^{-1}(y) \iff y = f(x). \quad (1)$$

Question 1 Suppose that f has an inverse f^{-1} , that $f(1) = 20$, and that $f^{-1}(30) = 2$. What are the values of $f^{-1}(20)$ and $f(2)$?

The domain of f^{-1} is the range of f and the range of f^{-1} is the domain of f .

Figures 3 and 4 illustrate how the domains and ranges of a function and its inverse are related. The domain of $y = f(x)$ in Figure 3 is the interval $a \leq x \leq b$ on the horizontal x -axis and its range is the interval $c \leq y \leq d$ on the vertical y -axis. The domain of $x = f^{-1}(y)$ in Figure 4 is the interval $c \leq y \leq d$ on the horizontal y -axis and its range is the interval $a \leq x \leq b$ on the vertical x -axis.

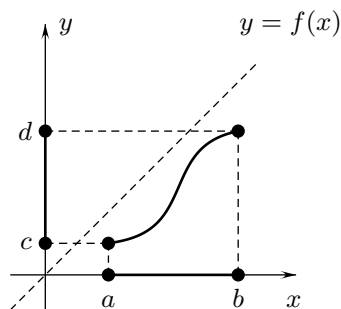


FIGURE 3

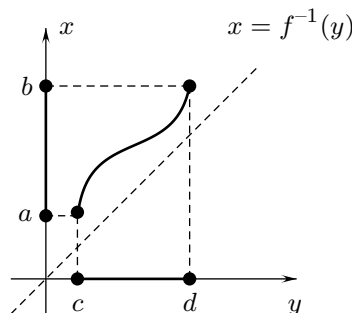


FIGURE 4

The function f and its inverse f^{-1} undo each other in the sense that

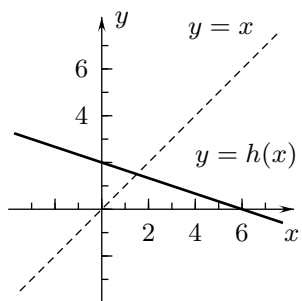
$$f^{-1}(f(x)) = x \quad \text{for } x \text{ in the domain of } f \quad (2)$$

$$f(f^{-1}(y)) = y \quad \text{for } y \text{ in the domain of } f^{-1}. \quad (3)$$

Example 1 Figure 5 shows the graph $y = h(x)$ of the function $h(x) = 2 - \frac{1}{3}x$. Find a formula for the inverse function $x = h^{-1}(y)$ and draw its graph in a yx -plane.

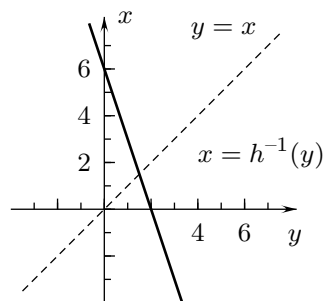
SOLUTION To find a formula for $x = h^{-1}(y)$, we solve $y = 2 - \frac{1}{3}x$ for x . We obtain first $\frac{1}{3}x = 2 - y$ and then $x = 6 - 3y$. Thus, the inverse function $x = h^{-1}(y)$ is given by $h^{-1}(y) = 6 - 3y$. Its graph in the yx -plane of Figure 6 is the line of slope -3 with y -intercept 6 and is the mirror image about the diagonal line of the graph of $y = h(x)$ in Figure 5. (The diagonal line that makes equal angles with the positive axes in Figures 5 and 6 is $y = x$ because the scales on the axes are equal.) \square

[†]The symbol \implies means “implies,” \impliedby means “is implied by,” and \iff means “if and only if.”



$$h(x) = 2 - \frac{1}{3}x$$

FIGURE 5



$$h^{-1}(y) = 6 - 3y$$

FIGURE 6

Changing variables

In Definition 1 and Example 1, the variable y of the inverse function $x = f^{-1}(y)$ was the letter used for values of the original function $y = f(x)$. In some cases it is convenient to have x be the variable of both the function and its inverse. A formula for the inverse function can then often be found by the following rule.

Rule 1 To find a formula with variable x for the inverse of $y = f(x)$, first solve the equation $y = f(x)$ to obtain the inverse $x = f^{-1}(y)$ with variable y . Then interchange the letters x and y to obtain a formula for $y = f^{-1}(x)$.

The procedure described in Rule 1 is illustrated in Figures 7 through 9. We begin with the function $y = f(x)$ in the xy -plane of Figure 7. We solve for x to obtain the inverse function $x = f^{-1}(y)$ with variable y in the xy -plane of Figure 8. Then we interchange the letters x and y to obtain the function $y = f^{-1}(x)$ with variable x in the xy -plane of Figure 9.[†]

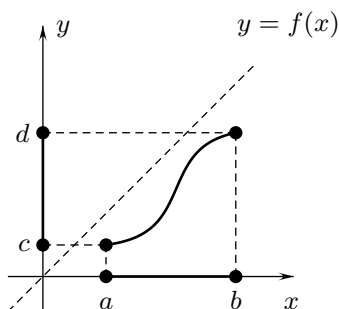


FIGURE 7

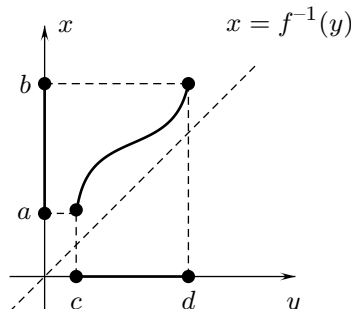


FIGURE 8

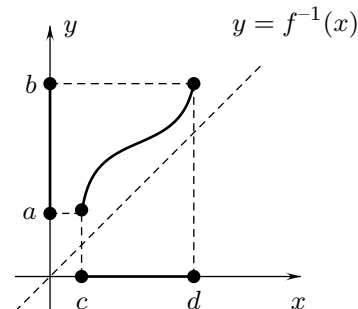


FIGURE 9

Question 2 How can Figure 9 be obtained directly from Figure 7?

[†]Notice that interchanging x and y to obtain $y = f^{-1}(x)$ from $x = f^{-1}(y)$ is not the same as solving the latter equation for y . That would take us back to the original equation $y = f(x)$. When we interchange x and y , we are not solving an equation, we are just interchanging the letters used for the variable and for values of the function.

Example 2 Figure 10 shows the graph of the function $y = x^3$ with the diagonal line that makes equal angles with the positive axes. Find a formula for the inverse of $y = x^3$ with x as variable and draw its graph.

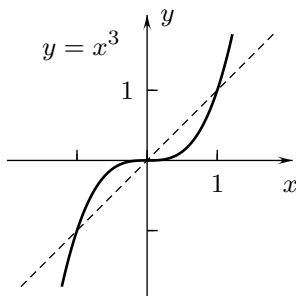


FIGURE 10

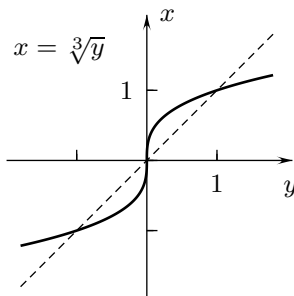


FIGURE 11

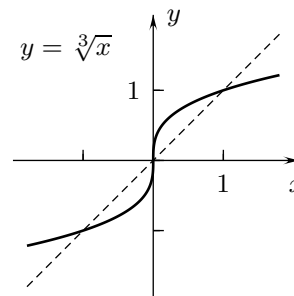


FIGURE 12

SOLUTION We solve $y = x^3$ by taking cube roots of both sides to find the inverse $x = \sqrt[3]{y}$ with y as variable. The graph of this function in the yx -plane of Figure 11 is obtained by flipping all of Figure 10 about the dashed line in that drawing. We interchange the letters x and y to obtain $y = \sqrt[3]{x}$. The graph of this function in the xy -plane of Figure 12 can be obtained by interchanging x and y in Figure 11 or by reflecting the curve in Figure 10 about the diagonal line without moving the axes. \square

The next two examples illustrate the fact that a function and its inverse undo each other if one is applied immediately after the other.

Example 3 Express the solutions x of $3h(x^2) = 12$ in terms of the function h^{-1} .

SOLUTION We want to use the fact that $h^{-1}(h(x)) = x$, so we first divide both sides of $3h(x^2) = 12$ by 3 to obtain $h(x^2) = 4$. Then, when we apply the function $y = h^{-1}(x)$ to both sides, we obtain $h^{-1}(h(x^2)) = h^{-1}(4)$, which simplifies to $x^2 = h^{-1}(4)$. Finally, taking square roots of both sides gives the solutions $x = \pm\sqrt{h^{-1}(4)}$. \square

Example 4 Express the solution x of $\sqrt{S^{-1}(x+1)} = 5$ in terms of the function S .

SOLUTION It would do us no good to apply the function S to the given equation because of the square root. Instead, we first square both sides to obtain $S^{-1}(x+1) = 25$. Applying S to both sides of this equation then gives $S(S^{-1}(x+1)) = S(25)$, which simplifies to $x+1 = S(25)$. We subtract 1 from both sides to obtain $x = S(25) - 1$. \square

Restricting domains

If a horizontal line $y = c$ intersects the graph of a function $y = f(x)$ at more than one point, then the function does not have an inverse, because the inverse function cannot have more than one value at $y = c$. The squaring function $y = x^2$ in Figure 13, for example, does not have an inverse because the horizontal line $y = c$ for any positive c intersects the graph at two points.

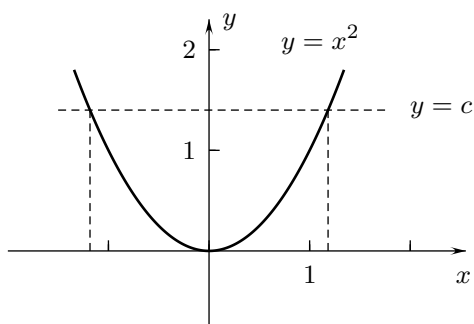
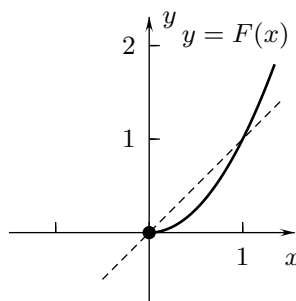


FIGURE 13



$$F(x) = x^2 \text{ for } x \geq 0$$

FIGURE 14

To obtain an inverse, we define F to be the function with values $y = x^2$ but defined only for $x \geq 0$ (Figure 14). Then the horizontal line $y = c$ for any $c \geq 0$ intersects the graph at only one point. The inverse of F with variable y is the square-root function $x = \sqrt{y}$ in Figure 15 and its inverse with variable x is $y = \sqrt{x}$ in Figure 16.

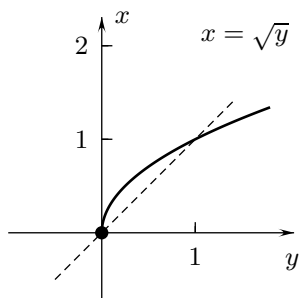


FIGURE 15

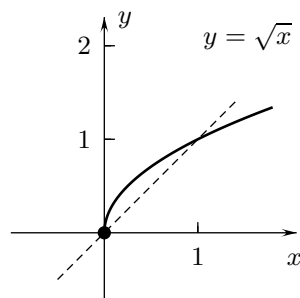


FIGURE 16

- Example 5**
- (a) Explain why the function $y = 3 - x^4$ of Figure 17 does not have an inverse.
 - (b) Draw the graph of the function $y = G(x)$ defined by $G(x) = 3 - x^4$ for $x \geq 0$.
 - (c) Find a formula for the inverse $y = G^{-1}(x)$ and draw its graph.

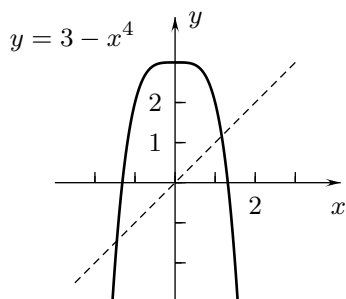
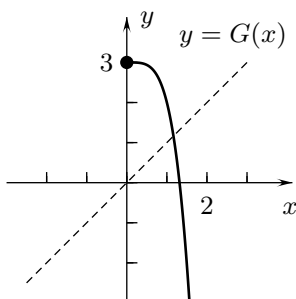
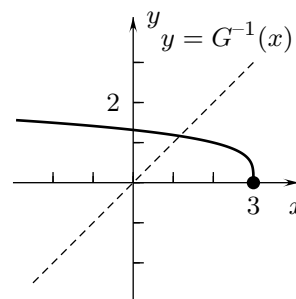


FIGURE 17



$$G(x) = 3 - x^4, x \geq 0$$

FIGURE 18



$$G^{-1}(x) = \sqrt[4]{3-x}$$

FIGURE 19

- SOLUTION**
- (a) The function $y = 3 - x^4$ of Figure 17 does not have an inverse because the horizontal line at any $y < 3$ intersects the graph at two points.
- (b) The graph of $y = G(x)$ in Figure 18 is the portion of $y = 3 - x^4$ for $x \geq 0$.
- (c) The equation $y = G(x)$ is equivalent to $y = 3 - x^4$ with $x \geq 0$. To solve for x , we subtract 3 from both sides of the equation to have $y - 3 = -x^4$ with $x \geq 0$. Multiplying both sides by -1 and taking fourth roots gives $x = \pm \sqrt[4]{3 - y}$ and then $x = \sqrt[4]{3 - y}$ since $x \geq 0$. Therefore, $G^{-1}(y) = \sqrt[4]{3 - y}$, and interchanging x and y yields $G^{-1}(x) = \sqrt[4]{3 - x}$.

The graph of $y = G^{-1}(x)$ in Figure 19 can be obtained by reflecting the graph of G in Figure 18 about the diagonal line at 45° to the positive x - and y -axes. \square

- Question 3** Figure 20 shows the graph of $y = H(x)$ defined by $H(x) = 3 - x^4$ for $x \leq 0$. Find a formula for $y = H^{-1}(x)$ and draw its graph.

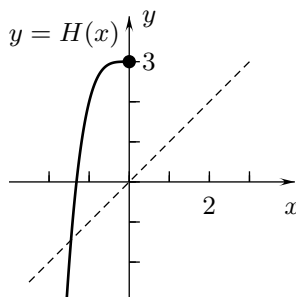


FIGURE 20

$$H(x) = 3 - x^4, x \leq 0$$

Logarithms

For any constant $b > 1$, the exponential function $y = b^x$ of Figure 21 has an inverse, which is called the LOGARITHM TO THE BASE b AND IS DENOTED $x = \log_b y$ (FIGURE 22). Thus,

$$y = b^x \quad \text{if and only if} \quad x = \log_b y. \quad (4)$$

Here x is the variable of the exponential function and y is the variable of the logarithm.

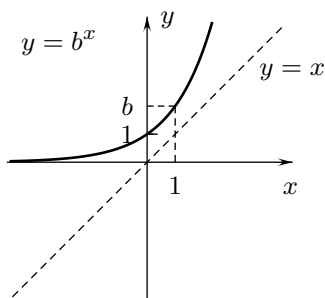


FIGURE 21

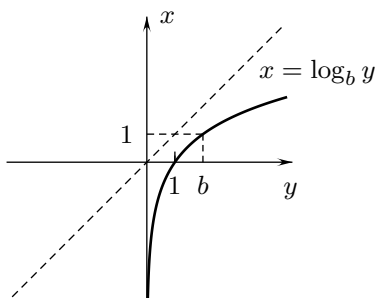


FIGURE 22

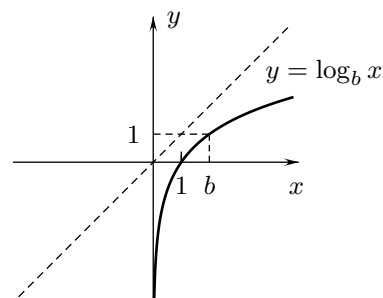


FIGURE 23

Interchanging x and y in Figure 22 gives the graph $y = \log_b x$ in Figure 23 of the logarithm with variable x . It is the mirror image of $y = b^x$ about the diagonal line in Figure 21.

Because $y = b^x$ is defined for all x and its range is the interval $y > 0$, $y = \log_b x$ is defined for $x > 0$ and its range is the set of all numbers y . The fact that $y = b^x$ and $y = \log_b x$ are inverse functions is also expressed in the two equations,

$$\log_b(b^x) = x \quad \text{for all } x \quad (5)$$

$$b^{\log_b x} = x \quad \text{for all } x > 0. \quad (6)$$

Notice that, while these formulas are visually quite complex, all they state is that $y = b^x$ and $y = \log_b x$ undo each other.

The basic properties of $\log_b x$ follow from properties of b^x discussed in the last section. For any $b > 1$, any n , and any positive x and y ,

$$\log_b(1) = 0 \quad (7)$$

$$\log_b(b) = 1 \quad (8)$$

$$\log_b(xy) = \log_b x + \log_b y \quad (9)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y \quad (10)$$

$$\log_b(x^n) = n \log_b x. \quad (11)$$

Also, logarithms with different bases, $b > 1$ and $c > 1$, are related by the formula

$$\log_b x = [\log_b c] [\log_c x] = \frac{\log_c x}{\log_c b}. \quad (12)$$

To remember formulas (12), notice that the x is higher the b in all three expressions and that one c is higher than the other in each of the last two expressions.

Example 6 Solve $2^x = 5$ for x .

SOLUTION $2^x = 5$ for $x = \log_2(5)$. \square

Question 4 Solve $\log_{10} y = -1$ for y .

The common logarithm

The logarithm to the base 10, $y = \log_{10} x$, is called the COMMON LOGARITHM AND IS OFTEN DENOTED $y = \log x$ WITH NO SUBSCRIPT. It was employed to simplify calculations involving products and powers before there were calculators and computers, and is still used in some applications.

Example 7 In chemistry the pH of a solution is defined by $pH = -\log_{10} [H^+]$, where $[H^+]$ is the concentration of hydrogen ions in the solution, measured in moles per liter. (1 mole = 6×10^{23} molecules.) The hydrogen ion concentration of water is 10^{-7} moles per liter. What is the pH of water?

SOLUTION The pH of water is $-\log_{10}(10^{-7}) = -(-7) = 7$. \square

The natural logarithm

The logarithm that is most convenient for calculus is $y = \log_e x$, with base equal to the base e of the natural exponential function $y = e^x$ that was introduced in the last section. This logarithm is called the NATURAL LOGARITHM and is usually denoted $y = \ln x$ (read “ell n x ”):

$$\ln x = \log_e x \quad \text{for all } x > 0. \quad (13)$$

Since $y = e^x$ and $y = \ln x$ are inverse functions,

$$e^{\ln x} = x \quad \text{for all } x > 0 \quad (14)$$

$$\ln(e^x) = x \quad \text{for all } x. \quad (15)$$

The graphs $y = e^x$ and $y = \ln x$ in Figure 24 are mirror images of each other about the line $y = x$ because there are equal scales on the axes in that drawing.

Example 8 Solve the equation $e^{3x} = 100$ for x .

SOLUTION We take the natural logarithm of both sides of the equation to obtain $\ln(e^{3x}) = \ln(100)$. Since the natural logarithm and exponential function are inverses, this gives $3x = \ln(100)$. Finally, we divide by 3 to obtain the solution $x = \frac{1}{3} \ln(100)$. \square

Question 5 Check the result of Example 8 by using a calculator or computer to find the approximate decimal value of the solution x and then to evaluate e^{3x} with this number.

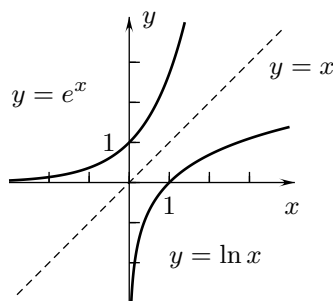


FIGURE 24

Example 9 Find all solutions x of $(\ln x)^2 = 1$.

SOLUTION We cannot use the fact that $y = \ln x$ and $y = e^x$ are inverses by applying the exponential function to both sides of the given equation because the logarithm is squared. Instead, we first take the square roots of both sides to have $\ln x = \pm 1$. Then applying the exponential function gives the solutions $x = e^1 = e$ and $x = e^{-1} = 1/e$. \square

Question 6 Generate the curve $y = (\ln x)^2$ with the line $y = 1$ in the window $-1 \leq x \leq 5$, $-0.5 \leq y \leq 1.5$ to illustrate the results of Example 9.

The natural logarithm can be used to find values of logarithms to any base $b > 1$ by using (12) with $c = e$:

$$\log_b x = \frac{\ln x}{\ln b} \quad \text{for all } x > 0. \tag{16}$$

To remember this formula, notice that the x is higher than the b on both sides of the equation.

Example 10 Give the approximate decimal value of $\log_2(3)$.

SOLUTION Formula (16) with $b = 2$ and $x = 3$ gives $\log_2(3) = \frac{\ln(3)}{\ln(2)} \doteq 1.58496250$. \square

Responses 0.4

Response 1 $f^{-1}(20) = 1$ because $f(1) = 20$ • $f(2) = 30$ because $f^{-1}(30) = 2$

Response 2 To obtain the graph of $y = f^{-1}(x)$ in Figure 9, reflect the graph of $y = f(x)$ in Figure 7 about the diagonal line $y = x$, without moving the x - and y -axes.

Response 3 Solve $y = 3 - x^4$ for x with $x \leq 0$. • $x^4 = 3 - y$ • $x = -\sqrt[4]{3 - y}$ • $H^{-1}(y) = -\sqrt[4]{3 - y}$ • $H^{-1}(x) = -\sqrt[4]{3 - x}$ • The graph of $y = H^{-1}(x)$ in Figure R3 is the mirror image about the dashed line of the graph of $y = H(x)$ in Figure 20.

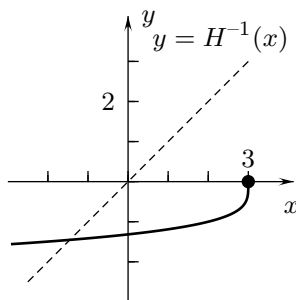


Figure R3

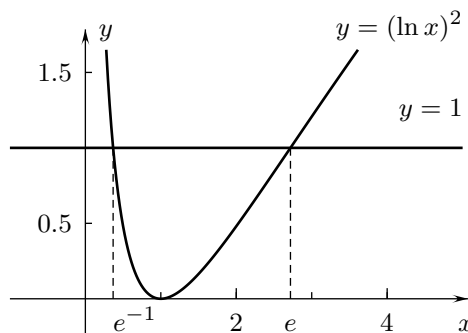


Figure R6

Response 4 $\log_{10} y = -1$ for $x = 10^{-1} = \frac{1}{10}$

Response 5 $x = \frac{1}{3} \ln(100) \doteq 1.535056729$ • $e^{3(1.535056729)} \doteq 100.0000001$

Response 6 Figure R6 shows the curve $y = (\ln x)^2$, the line $y = 1$, and their intersections, which are at $x = e \doteq 2.71$ and $x = e^{-1} \doteq 0.37$.

Interactive Examples 0.4

Interactive solutions are on the web page <http://www.math.ucsd.edu/~ashenk/>.[†]

- Find approximate values of (a) $G(4)$, (b) $G^{-1}(10,000)$, and (c) $G^{-1}(G(3) + 3000)$, where G is the function whose graph is in Figure 25.

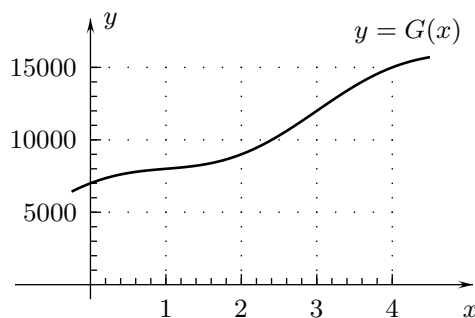


FIGURE 25

- Solve $e^{x^4} = 1000$ for x . Then generate $y = e^{x^4}$ and $y = 1000$ in the window $-2.5 \leq x \leq 2.5$, $-150 \leq y \leq 1500$ and use an intersect or trace operation to check your answer.
- Solve $2^{x+1} = 2^{x^2+x}$ for x . Then generate $y = 2^{x+1}$ and $y = 2^{x^2+x}$ in the window $-2 \leq x \leq 2$, $-1 \leq y \leq 7$, and use an intersect or trace operation to check your answer.
- Solve $-2 \ln(x^5) = 12$ for x .
- Solve $4^x = 5(8^x)$ for x .
- Solve $[\ln(5x)]^2 = 9$ for x .
- Solve $F^{-1}(F(F^{-1}(x))) = -2$ under the assumption that $y = F(x)$ and its inverse $y = F^{-1}(x)$ are defined for all x .

Exercises 0.4

^A Answer provided. ^O Outline of solution provided. ^C Graphing calculator or computer required.

CONCEPTS:

- Simplify the expressions (a) $f(f^{-1}(f(x)))$ and (b) $f^{-1}(f(f^{-1}(x)))$, where x is a number in the domains of f and f^{-1} .
- (a) What are the values of $\log_{10}(100)$, $\log_{10}(1000)$, and $\log_{10}(100,000)$? (b) How are the numbers from part (a) related and what property of logarithms is illustrated by this relationship?
- (a) What are the values of $\ln(e^3)$, $\ln(e^5)$, and $\ln(e^3 e^5)$? (b) How are the numbers from part (a) related and what property of logarithms is illustrated by this relationship?
- (a) What are the values of $3 \ln(e^5)$, and $\ln((e^5)^3)$? (b) How are the numbers from part (a) related and what property of logarithms is illustrated by this relationship?

[†]In the published text the interactive solutions of these examples will be on an accompanying CD disk which can be run by any computer browser without using an internet connection.

BASICS:

5. Express the solutions x of (a) $f(x) = 4$ and (b) $f^{-1}(x) = -7$ in terms of values of f and f^{-1} .
- 6.⁰ What are (a) $P(2)$, (b) $P^{-1}(0.2)$, and (c) $P(P^{-1}(0.2) + 2)$ if P is given by the following table?

x	0	1	2	3	4	5
$P(x)$	4.1	3.9	2.1	0.2	-1.4	-3

- 7.⁰ (a) Find a formula for $y = f^{-1}(x)$ where $f(x) = x^{11/3}$. (b) Generate the graphs of $y = f(x)$ and $y = f^{-1}(x)$ with the line $y = x$ in an xy -plane and copy them on your paper. Use a window with equal scales on the axes that includes the square $-2 \leq x \leq 2, -2 \leq y \leq 2$.
- 8.⁰ Solve (a) $\log_{10}(x^2) = -4$, (b) $10^{2x} = 5$, (c) $\sqrt{\ln x} = 2$, and (d) $e^{\sqrt{x}} = 7$ for x . Give exact answers and approximate decimal values. Check your answers with a calculator or computer.
- 9.⁰ Use properties of the common logarithm to show that each of the numbers (a)–(c) in the first column below equals one of the numbers (I)–(III) in the second column.

(a) $\log_{10}(300)$	(I) $2\sqrt{\log_{10}(3)}$
(b) $\log_{10}(30) - \log_{10}(3)$	(II) $2 + \log_{10}(3)$
(c) $\sqrt{\log_{10}(81)}$	(III) 1

- 10.⁰ Figure 26 shows the graph of a function $y = H(x)$, defined for $0 \leq x \leq 6$. Draw the graph of its inverse $y = H^{-1}(x)$.

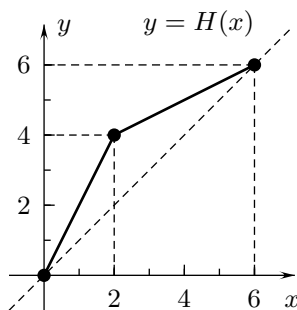


FIGURE 26

11. The graph of $y = k(x)$ is in Figure 27. Draw the graph of its inverse $y = k^{-1}(x)$.

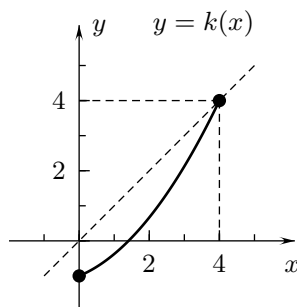


FIGURE 27

- 12.⁰** The function $r = G(v)$ of Figure 28 gives the rate of gasoline consumption of a car, measured in miles per gallon, as a function of the car's velocity v , measured in miles per hour. **(a)** What is the action of the inverse function $v = G^{-1}(r)$? **(b)** What is approximate value of $G^{-1}(15)$?

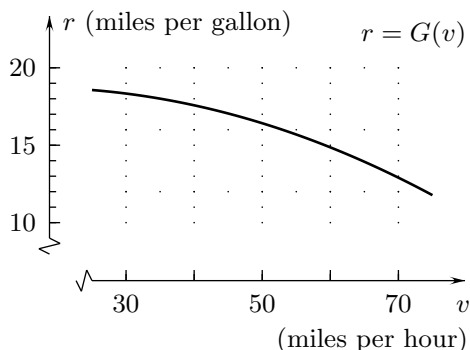


FIGURE 28

- 13.⁰** Figure 29 shows the graph of the function $P = F(Q)$ that converts the number of quarts Q in a volume into the number of pints P . **(a)** What is the action of $Q = F^{-1}(P)$? **(b)** Give formulas for $P = F(Q)$ and $Q = F^{-1}(P)$. **(c)** Draw the graph of $Q = F^{-1}(P)$ in a PQ -plane

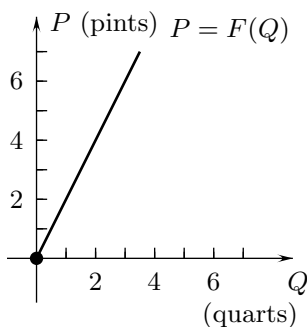


FIGURE 29

- 14.^A** An airplane that is flying west at the constant speed of 500 miles per hour flies $s = f(t)$ miles in t (hours) for $t \geq 0$, where $f(t) = 500t$. **(a)** What is the action of $t = f^{-1}(s)$? **(b)** Find a formula for $t = f^{-1}(s)$. Then draw the graph $s = f(t)$ in a ts -plane and the graph of $t = f^{-1}(s)$ in an st -plane.
- 15.** A grain silo contains 200 tons of wheat at the beginning of an eight-hour work day. During the day wheat is removed at the constant rate of 10 tons per hour, so that the weight of wheat in the silo t hours after the beginning of the work day is $w = F(t)$ tons, where $F(t) = 200 - 10t$ of **(a)** What does $F^{-1}(100)$ represent in terms of weight and time? **(b)** Give a formula for $t = F^{-1}(w)$.
- 16.^A** The volume of a sphere of radius r is $V = G(r)$ with $G(r) = \frac{4}{3}\pi r^3$ for $r \geq 0$. Give a formula for $r = G^{-1}(V)$. What does this function do?
- 17.** If V gallons of a salt solution contains 5 pounds of salt, then the concentration of the solution is $y = f(V)$ pounds per gallon, where $f(V) = 5/V$. **(a)** What does the inverse function $V = f^{-1}(y)$ do? **(b)** Find a formula for $V = f^{-1}(y)$. **(c)** Draw the graph of $y = f(V)$ in a Vy -plane and the graph of $V = f^{-1}(y)$ in a yV -plane. Use $-1 \leq V \leq 8$ and $-1 \leq y \leq 8$ and have equal scales on the axes in both drawings.
- 18.⁰** **(a)** Find a formula for the inverse of $y = \sqrt[3]{x+1}$ as a function of x . **(b)** Generate the graphs of both functions with $y = x$ in a window that includes $-3 \leq x \leq 3$, $-3 \leq y \leq 3$ and has equal scales on the axes. Copy the drawing on your paper.[†]

[†]If your calculator or computer does not generate the graph of $\sqrt[3]{x+1}$ for $x < -1$, enter it as the two functions $(x+1)^{(1/3)}$ for $x \geq -1$ and $-(\text{abs}(x+1))^{(1/3)}$ for $x < -1$. Use the setting ZSquare under Zoom to get equal scales on a TI calculator.

In Exercises 37 through 39 **(a)** solve the equations for x . **(b)** Generate the graphs of the functions on both sides of the equations in suitable windows to check your answers.

37.^A $\ln(1 + x^2) = -1$

38.^A $x^{x-2} = 1$

39. $2^{x^2} = 16(2^{-x^2})$

40. The magnitude of an earthquake on the Richter scale is $M = 0.67 \log_{10}(0.37E) + 12.46$, where E is the energy of the earthquake in kilowatt hours. What is the energy **(a^A)** of an earthquake of magnitude 5 and **(b)** of an earthquake of magnitude 6? Give approximate decimal values.

41. The loudness of sound can be measured in DECIBELS, where one decibel is supposed to be the smallest change in loudness that the average human can detect. (One BEL, named in honor of Alexander Graham Bell equals ten decibels.) Sound reaches the eardrum as an oscillation in the air pressure. If the variation in air pressure is P pounds per square inch, then the loudness of the sound is $D = 20 \log_{10}[(3.45 \times 10^8)P]$ decibels. What is the variation in sound pressure caused by **(a)** a whisper at 10 decibels and **(b)** a rock band at 120 decibels?

42. The MAGNITUDE of a star is defined by the formula $M = -2.5 \log_{10}(kI)$, where k is a positive constant and I is the intensity of the light from the star. **(a)** Do brighter stars have less or greater magnitudes than dimmer stars? **(b)** What is the ratio of the intensities of light from the brightest star Sirius, which has a magnitude of -1.6 and from the star Betelgeuse, which has a magnitude of 0.9 ?

In Exercises 43 through 46 solve the equations for x under the assumption that the functions and their inverses are defined for all x .

43.^O $\frac{4}{G(x)} = G(x)$

45. $[R(x)]^2 - 2R(x) + 1 = 0$

44.^A $H(x)H^{-1}(x) = 0$

46. $\frac{3S^{-1}(x)}{S^{-1}(x) + 1} = 6$

(End of Section 0.4)