## Section 14.R

(3.24/08)

## Review exercises

**1.**[A] What is the value of $f(x,y) = x^2e^y + 2\cos y$ at $(5,0)$?

**2.**[A] Find $g(10,-2)$ for $g(x,y) = x^2 - 10y^3$.

**3.**[A] Draw the level curves $h = c$ of the linear function $h(x,y) = y - 2x$ for $c = 0, \pm2, \pm4, \pm6$.

**4.**[A] Draw the level curves $F = c$ of the function $F(x,y) = \left(y - \frac{1}{2}x^2\right)^3$ for $c = 0, \pm1, \pm8$.

**5.**[A] Draw the level curve of $f(x,y) = \dfrac{y}{\cos x}$ through the point $(0,2)$. What is value of $f$ on this curve?

Find the $x$- and $y$-derivatives of the functions in Exercises 6 through 11.

**6.**[A] $z = x^2y^3 - 3x^2y$

**7.**[A] $z = \sin x + \cos y$

**8.**[A] $z = x^2e^{3y}$

**9.**[A] $f(x,y) = (1+xy)^5$

**10.**[A] $g(x,y) = x\ln y - y\ln x$

**11.**[A] $h(x,y) = e^{2x+3y}$

**12.**[A] What are $q_x(0,0)$ and $q_y(0,0)$ if $q(x,y) = \sqrt{Q(x,y)}$, $Q(0,0) = 9, Q_x(0,0) = 10$, and $G_y(0,0) = 5$?

**13.**[A] What is the $t$-derivative of $H(t) = h(t^2,t^3)$ at $t = 1$ if $h(1,1) = 5, h_x(1,1) = 3$ and $h_y(1,1) = -4$?

**14.**[A] What are **(a)** $F(1,2)$, **(b)** $F_u(1,2)$, and **(c)** $F_v(1,2)$ if $F(u,v) = f(u^2v^3, 3u+7v)$ and $z = f(x,y)$ satisfies $f(8,17) = 10$, $f_x(8,17) = 5$ and $f_y(8,17) = -1$?

**15.**[A] What are **(a)** $G(0,2)$, **(b)** $G_u(0,2)$, and **(c)** $G_v(0,2)$ if $G(u,v) = g(v\sin u, u\sin v)$ and $z = g(x,y)$ satisfies $g(0,0) = 4, g_x(0,0) = 10$ and $g_y(0,0) = 2$?

**16.**[A] What is $H_u(3,5)$ for $H(u,v) = h(x(u,v), y(u,v))$ if $x(3,5) = 10, y(3,5) = 20$, $h_x(10,20) = -5, h_y(3,20) = 7, x_u(3,5) = 5$ and $y_u(3,5) = 2$?

**17.**[A] Find the directional derivative of $f(x,y) = x^2e^{3y}$ at $(2,1)$ in the direction toward $(5,5)$.

**18.**[A] What is the directional derivative of $z = \cos(xy)$ at $(2,3)$ in the direction toward the origin?

**19.**[A] Find the gradient of $f(x,y) = x^3y^2$ at $(2,3)$.

**19.**[A] What is the gradient of $g(x,y) = \sin x + \cos y$ at $(\frac{1}{6}\pi, \frac{1}{3}\pi)$?

**20.**[A] What is the maximum directional derivative of $f(x,y) = x^2 + y^3$ at $(1,1)$ and what is the unit vector in the direction of the maximum directional derivative?

**22.**[A] What is the minimum directional derivative of $f(x,y) = e^{x-y}$ at $(3,3)$ and what is the unit vector in the direction of the minimum directional derivative?

**23.**[A] Give the unit vectors in the directions in which the directional derivative of $z = x^2 - y^3$ at $(2,1)$ is zero.

**24.**[A] Give an equation of the tangent plane to $z = x^2 - xy$ at $x = 2, y = 3$.

**25.**[A] Give an equation of the tangent plane to $z = \sin(x+y)$ at $x = -2, y = 2$.

**26.**[A] Give the differential of $g(x,y) = x^2 - y^4$ at $(3,-1)$.

**27.**[A] What is the differential of $z = e^{xy}$ at $(-3,-4)$?

**28.**[A] Find the first-order partial derivatives of $w = 3x + 4y^2 - 5z^3$.

**29.**[A] What is $f_{xy}(1,1,1)$ for $f(x,y,z) = x^2y^3z^4$?

**30.**[A] Give a normal vector and an equation for the tangent plane to $xyz = 6$ at $(1,2,3)$.

**31.**[A] Give a normal vector and an equation for the tangent plane to $\sin x + \sin y + \sin z = 3$ at $(\frac{1}{2}\pi, \frac{1}{2}\pi, \frac{1}{2}\pi)$.

**32.**[A] What is the differential of $z = x^2 - y^3 + 5z$ at $(3,2,1)$?

**33.**[A] Find the maximum directional derivative of $h(x, y, z) = e^{x+y+z}$ at $(1, 2, 3)$ and the unit vector in the direction of that maximum derivative.

**34.**[A] The gradient of $w = f(x, y, z)$ at $(5, 10, 15)$ is $\langle 2, 3, 4\rangle$. What is the minimum directional derivative of $f$ at $(5, 10, 15)$?

Find the derivatives in Exercises 35 through 38.

**35.**[A] $\dfrac{\partial}{\partial w}(xy^2z^3w^4)$

**36.**[A] $\dfrac{\partial}{\partial y}(\sin x + \sin y + \sin z + \sin w)$

**37.**[A] $F_q(1, 1, 1, 1)$ for $F(p, q, r, s) = pq^2r^3s^4$

**38.**[A] $\dfrac{\partial}{\partial x_4}(x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5)$

**(End of Section 14.R)**