Section 1.R

(3/24/08)

Review exercises

1.[A] What is the limit of $\dfrac{x^2+4}{x+6}$ as $x \to 2$ and why?

2.[A] What is the limit of $1 + x + x^2 + x^3$ as $x \to 10$ and why?

3.[A] **(a)** Sketch the graph of

$$g(x) = \begin{cases} x^2 & \text{for} \quad x < 1 \\ 2 - x^{-2} & \text{for} \quad x > 1. \end{cases}$$

Find **(b)** $g(1)$, **(c)** $\lim_{x \to 1} g(x)$ and **(d)** $\lim_{x \to 2} g(x)$. **(e)** Solve $g(x) = 1$ for x.

4.[A] **(a)** Sketch the graph of

$$k(x) = \begin{cases} 2 + x & \text{for} \quad x < -1 \\ 1 & \text{at} \quad x = -1 \\ x^2 & \text{for} \quad x > -1. \end{cases}$$

Find **(b)** $k(-1)$ and **(c)** $\lim_{x \to -1} k(x)$. **(d)** At what points is $y = k(x)$ continuous?

5.[A] Without drawing the graph, find **(a)** $f(-2)$, **(b)** $f(-1)$, **(c)** $\lim_{x \to -1^-} f(x)$, **(d)** $\lim_{x \to -1^+} f(x)$, and
(e) $\lim_{x \to -1} f(x)$, where

$$f(x) = \begin{cases} x^2 - 2 & \text{for} \quad x < -1 \\ 0 & \text{for} \quad x = -1 \\ x^5 & \text{for} \quad x > -1. \end{cases}$$

6.[A] Without drawing the graph, find **(a)** $h(-6)$, **(b)** $h(3)$, **(c)** $\lim_{x \to 3^-} h(x)$, **(d)** $\lim_{x \to 3^+} h(x)$, and **(e)** $\lim_{x \to 3} h(x)$, where

$$h(x) = \begin{cases} 6/x & \text{for} \quad x < 3 \\ 10 & \text{for} \quad x = 3 \\ x^2 - 5 & \text{for} \quad x > 3. \end{cases}$$

C **7.**[A] Predict the limit of $y = \dfrac{x^4 - 16}{4 - x^2}$ as $x \to 2$ by calculating values for x close to 2.

C **8.**[A] Predict the limit of $y = \dfrac{x - 4}{\sqrt{x} - 2}$ as $x \to 4$ by calculating values for x close to 2.

9.[A] Find $\lim_{x \to 10} [x^2 P(x)]$, where $\lim_{x \to 10} P(x) = 5$.

10.[A] Find $\lim_{x \to 4} \left(\dfrac{Q(x) + x^2}{Q(x) - x} \right)$, where $\lim_{x \to 4} Q(x) = 2$.

11.[A] Find $\lim_{x \to -1} \dfrac{x^2 - x - 2}{x + 1}$.

12.[A] Find $\lim_{x \to 3} \dfrac{x^2 - 6x + 9}{x - 3}$.

13.[A] Find $\lim_{x \to 1} \dfrac{2x - 2}{x^2 - x}$.

In Exercises 14 through 16 use the formulas to find **(a)** $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

14.[A] $f(x) = 5 + x - x^3$

15.[A] $f(x) = \dfrac{x^3 + x}{x^4 - x^2 + 1}$

16.[A] $f(x) = \dfrac{1 - 3x^4}{x^4 - 5x^3}$

In Problems 17 through 19 use the formulas to find **(a)** $\lim_{x\to\infty} f(x)$, **(b)** $\lim_{x\to-\infty} f(x)$, **(c)** $\lim_{x\to 0^+} f(x)$, **(d)** $\lim_{x\to 0^-} f(x)$, and **(e)** $\lim_{x\to 0} f(x)$.

17.[A] $f(x) = x^3 - 1/x^3$

18.[A] $f(x) = 5 - 6/x$

19.[A] $f(x) = -3 - 1/x^2$

20.[A] Find $\lim_{x\to 3^+} \dfrac{x^2}{x-3}$ and $\lim_{x\to 3^-} \dfrac{x^2}{x-3}$.

21.[A] Find $\lim_{x\to 2} \dfrac{3-2x}{(x-2)^2}$.

22.[A] What are the intervals on which which the function $y = S(x)$ of Figure 1 is continuous?

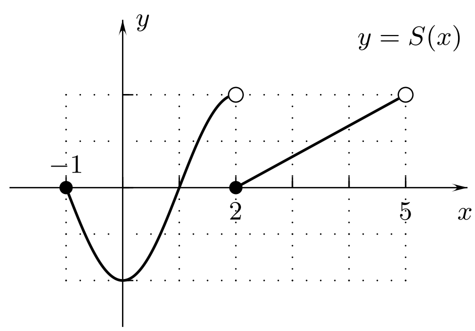

FIGURE 1

23.[A] What are the intervals on which which $y = \cos\left(\dfrac{1}{x-1}\right)$ is continuous?

24.[A] What are the intervals on which $y = \sqrt{x+5}$ is continuous?

25.[A] What are the intervals on which which $y = \dfrac{1}{\sqrt{x}} - \sqrt[3]{x-4}$ is continuous?

26.[A] What are the intervals on which which $y = \ln\left(\dfrac{1}{x^2+1}\right)$ is continuous?

27.[A] What is the limit of $y = \ln(x+10)$ as $x \to -5$ and why?

28.[A] What is the limit of $y = x + \sqrt{x-1}$ as $x \to 1^+$ and why?

29.[A] Find $\lim_{x\to 0} F(x)$, where

$$F(x) = \begin{cases} e^{x^2} & \text{for} \quad x < 0 \\ 5 & \text{for} \quad x = 0 \\ \cos(\sqrt{x}) & \text{for} \quad x > 0. \end{cases}$$

30.[A] Find $\lim_{x\to 0} G(x)$, where

$$G(x) = \begin{cases} \sqrt[3]{x+1} & \text{for} \quad x < 0 \\ 0 & \text{for} \quad x = 0 \\ \sqrt[3]{x+8} & \text{for} \quad x > 0. \end{cases}$$

Use Definitions 1 or 2 in Section 1.4 to verify the limits in Exercises 31 through 35.

31.[A] $\lim_{x\to 2} x^2 = 4$

32.[A] $\lim_{x\to 3} (x^2 - x + 1) = 7$

33.[A] $\lim_{x\to 0} \dfrac{x}{x^2+1} = 0$

34.[A] $\lim_{x\to 2^+} \dfrac{1}{(x-2)^3} = \infty$

35.[A] $\lim_{x\to 10^+} \dfrac{1}{\sqrt{x-10}} = \infty$

(End of Section 1.R)