

The ancient Greeks did some amazing mathematics. Their work on the theory of proportions, in plane geometry, and on tangent lines, areas, and volumes came close to meeting modern standards of exposition and logic. Their explanations of topics from physics, such as the motion of projectiles and falling bodies, in contrast, were very different from modern theories. The philosopher Aristotle (384–322 BC), for example, said that the motion of projectiles is caused by air pushing them from behind and that bodies fall with speeds proportional to their weights.⁽¹⁾ His point of view was generally not questioned in Western Europe until the Renaissance, when calculus was developed to study rates of change of nonlinear functions that arise in the study of motion and Newton explained how forces such as air resistance and gravity affect motion. We see in this chapter that the rate of change of a function is its DERIVATIVE, which is the slope of a tangent line to its graph. We study linear functions and constant rates of change in Section 2.1 and average rates of change in Section 2.2. In Section 2.3 we give the general definition of the derivative as a limit of average rates of change. Formulas for exact derivatives of powers of x and of linear combinations of functions are derived in Section 2.4. In Section 2.5 we look at derivatives as functions and discuss applications that require finding approximate derivatives from graphs and tables. Rules for finding derivatives of products, quotients, and powers of functions are discussed in Sections 2.6 and 2.7. Section 2.8 deals with linear approximations and differentials.

Section 2.1

Linear functions and constant rates of change

OVERVIEW: As we will see in later sections, the rates of change of most functions are found by using calculus techniques to find their derivatives. In this section, however, we study applications that involve a special class of functions whose rates of change can be determined without calculus. These are the LINEAR FUNCTIONS. A linear function is a first-degree polynomial $y = mx + b$. Its graph is a line and its (constant) rate of change is the slope of its graph.

Topics:

- *Constant velocity*
- *Other constant rates of change*
- *Approximating data with linear functions*

Constant velocity

If an object moves on a straight path, we can use an s -axis along that path, as in Figure 1, to indicate the object's position. The object's s -coordinate is then a function of the time t , which is linear if the object's velocity is constant.

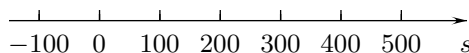


FIGURE 1

Example 1 A moving van is 200 miles east of a city at noon and is driving east at the constant velocity of 60 miles per hour. Give a formula for the van's distance $s = s(t)$ east of the city t hours after noon.

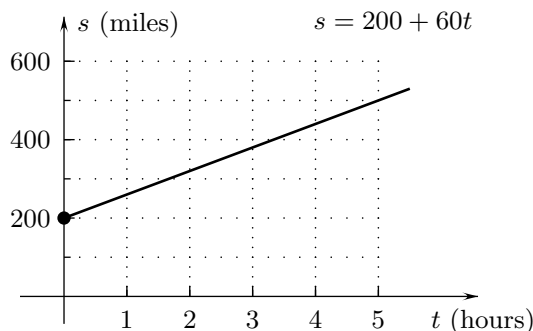
SOLUTION After t hours the van has traveled $\left[t \text{ hours} \right] \left[60 \frac{\text{miles}}{\text{hour}} \right] = 60t$ miles, so it is $s = 200 + 60t$ miles east the city. \square

⁽¹⁾ *Encyclopedia of Cosmology* by N Hetherington, New York: Garland Publishing, Inc, 1993, pp. 20–25.

The graph of the van's distance from the city is the line $s = 200 + 60t$ in the ts -plane of Figure 2.[†] Its slope is the velocity, 60 miles per hour, of the van. This illustrates the following basic principle.

$$\begin{aligned} [\text{Velocity}] &= [\text{Slope}] \\ &= 60 \text{ miles per hour} \end{aligned}$$

FIGURE 2

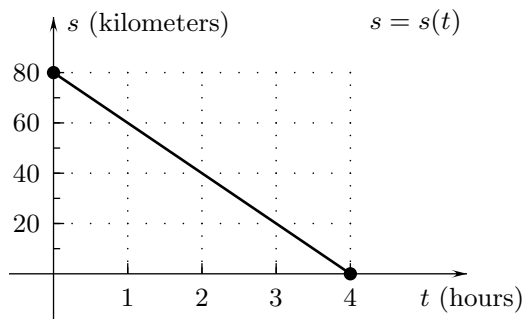


Definition 1 (Constant velocity) If an object moves at a constant velocity along an s -axis, then its position s is a linear function of the time t . The slope of the graph of this function is the object's velocity in the positive s -direction.

The units used for the velocity in Definition 1 are those used for distance divided by those used for time.

Example 2 Figure 3 shows the graph of a linear function that gives a bicyclist's distance $s = s(t)$ (kilometers) north of a restaurant at time t (hours) for $0 \leq t \leq 4$. What is the bicyclist's velocity toward the north?

FIGURE 3



SOLUTION The bicyclist's velocity toward the north is the slope $\frac{s(4) - s(0)}{4 - 0} = \frac{0 - 80}{4} = -20$ (kilometers per hour) of the line in Figure 3. (It is negative because the bicyclist is going south.) \square

Other constant rates of change

We can think of velocity as the rate of change of distance with respect to time. Then Definition 1 of constant velocity is an example of the following general definition of general constant rates of change:

Definition 2 (Constant rates of change) The (constant) RATE OF CHANGE with respect to the variable x of a linear function $y = f(x)$ is the slope of its graph.

If x and f have units in Definition 2, then the units of the rate of change are those of f divided by those of x .

[†]Notice that the equation $s = 200 + 60t$ is in the SLOPE-INTERCEPT form of an equation of a line.

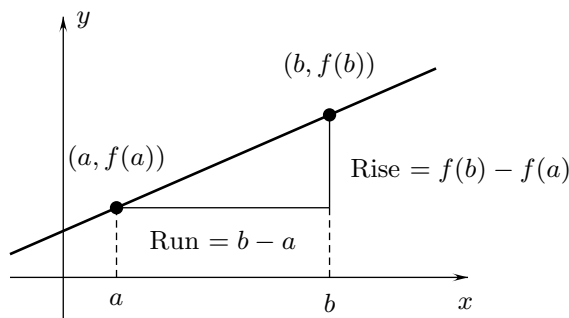
Since the rate of change of a linear function f is the slope of its graph, it can be calculated as the ratio,

$$[\text{Rate of change of } f] = \frac{f(b) - f(a)}{b - a}$$

of the rise and the run between any two points on the graph (Figure 4).

$$\begin{aligned} [\text{Rate of change}] &= [\text{Slope}] \\ &= \frac{f(b) - f(a)}{b - a} \end{aligned}$$

FIGURE 4



Example 3 The circumference C of a circle of radius r meters is $C = 2\pi r$ (meters). What is the rate of change of the circumference with respect to the radius?

SOLUTION The graph of the circumference is the line $C = 2\pi r$ in Figure 5. The rate of change of C with respect to r is the slope 2π (meters per meter) of that line. \square

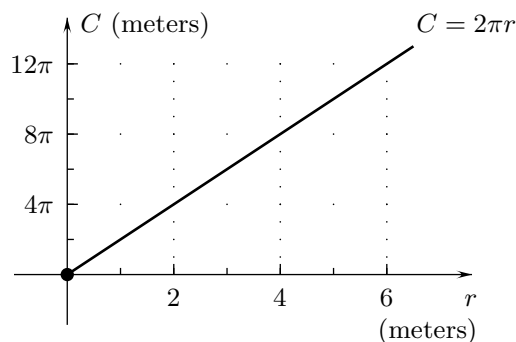


FIGURE 5

Example 4 A woman pays \$250 overhead per day to rent space and hire help for a popcorn stand. It costs her an additional \$7 per pound to make the popcorn and she sells it for \$32 per pound. What are her (a) revenue $R = R(x)$, (b) cost $C = C(x)$, and (c) profit $P = P(x)$ if she makes and sells x pounds in a day?

SOLUTION (a) Her revenue on x pounds of popcorn sold in a day is

$$R(x) = \left[x \text{ pounds} \right] \left[32 \frac{\text{dollars}}{\text{pound}} \right] = 32x \text{ dollars.} \quad (1)$$

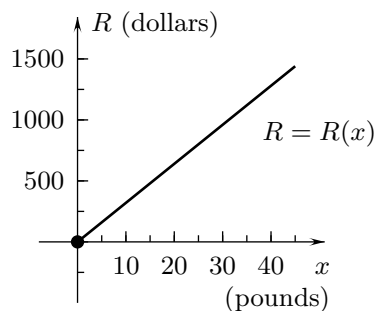
(b) Because she pays \$250 plus \$7 per pound of popcorn, the cost for x pounds is

$$C(x) = 250 + \left[x \text{ pounds} \right] \left[7 \frac{\text{dollars}}{\text{pound}} \right] = 250 + 7x \text{ dollars.} \quad (2)$$

(c) The profit is equal to the revenue (1), minus the cost (2) or

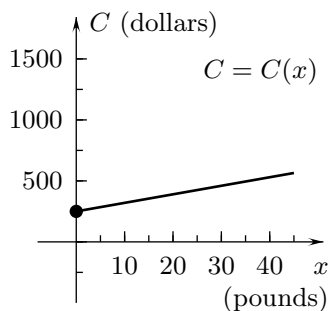
$$P(x) = R(x) - C(x) = 25x - 250 \text{ dollars.} \quad \square \quad (3)$$

The revenue, cost, and profit functions from Example 3 are the linear functions whose graphs are the lines in Figures 6 through 8. The rate of change of the revenue R with respect to x is the slope, 32 dollars per pound, of the first line; the rate of change of the cost C with respect to x is the slope, 7 dollars per pound, of the second line; and the rate of change of the profit P with respect to x is the slope, 25 dollars per pound, of the third line.



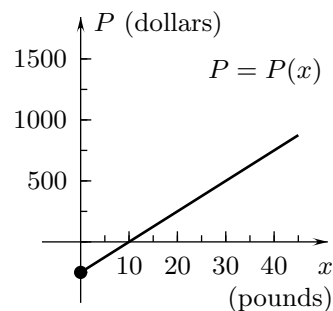
$$R = 32x$$

FIGURE 6



$$C = 250 + 7x$$

FIGURE 7



$$P = 25x - 250$$

FIGURE 8

Example 5 Imagine that you start on a trip with 15 gallons of gas in your tank, that your car uses gasoline at the constant rate of 0.05 gallons per mile (20 miles per gallon), and that you drive with the constant velocity of 60 miles per hour until your tank is empty. (a) Give a formula for the volume V (gallons) of gasoline remaining in your tank as a function of the distance s (miles) that you have gone on your trip. (b) Give a formula for the distance s (miles) as a function of the amount of time t (hours) you have driven. (c) Use the results of parts (a) and (b) to give a formula for the volume V of gasoline in your tank as a function of t . Draw the graphs of the three functions.

SOLUTION (a) Because you start with 15 gallons in your tank and use 0.05 gallons per mile, your tank contains

$$V = 15 - \left[s \text{ miles} \right] \left[0.05 \frac{\text{gallons}}{\text{mile}} \right] = 15 - 0.05s \text{ gallons}$$

when you have driven s miles. The tank is empty when $V = 15 - 0.05s$ is zero, which is at $s = 15/0.05 = 300$ miles, so the domain of V as a function of s is the interval $0 \leq s \leq 300$ and its graph is the line in Figure 9.

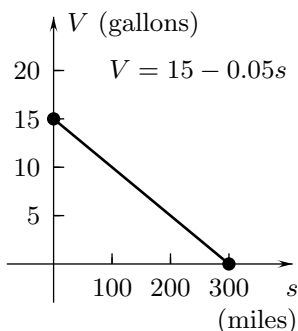


FIGURE 9

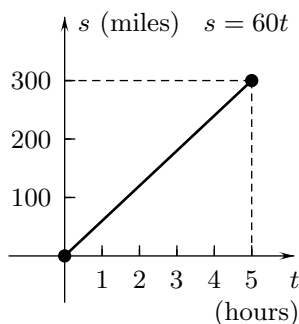


FIGURE 10

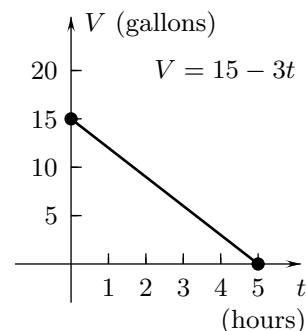


FIGURE 11

(b) Because you are driving 60 miles per hour, you travel

$$s = [t \text{ hours}][60 \text{ miles per hour}] = 60t \text{ miles}$$

in t hours. You run out of gas at 300 miles, which at $t = 300/60 = 5$ hours, so the domain of s as a function of t is the interval $0 \leq t \leq 5$ and its graph is in Figure 10.

(c) Since $V = 15 - 0.05s$ and $s = 60t$,

$$V = 15 - 0.05(60)t = 15 - 3t \text{ gallons.}$$

The graph of this function is the line in Figure 11. \square

The slopes in Figures 9 through 11 are all rates of change. The slope of the line $V = 15 - 0.05s$ in Figure 9 is the rate of change -0.05 (gallons per mile) of the volume of gas in your tank with respect to s . It is negative because the volume of gasoline in the tank decreases as the distance increases. The slope of the line $s = 60t$ in Figure 10 is your velocity; it is the rate of change 60 (miles per hour) of the distance you have traveled with respect to time. The slope of $V = 15 - 3t$ in Figure 11 is the rate of change -3 (gallons per hour) of the volume of gas in your tank with respect to time.

Example 6 A six-foot tall man is walking away from a fifteen-foot high lamppost. (a) Find a formula for the length y of his shadow as a function of his distance x from the lamppost, using the mathematical model of two right triangles in Figure 12, where the lamppost and the man form the vertical sides of the triangles and the horizontal ground is their base. (b) What is the rate of change of y with respect to x ?

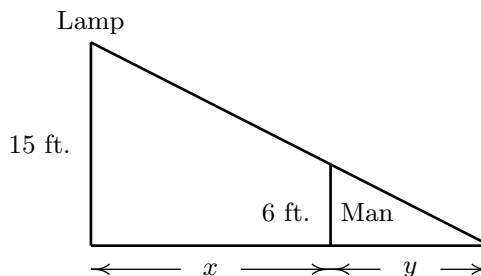


FIGURE 12

SOLUTION (a) Because the triangles are similar,[†] the length $x + y$ of the base of the larger triangle divided by its height 15 equals the length y of the base of the smaller triangle divided by its height 6: $\frac{x + y}{15} = \frac{y}{6}$. Multiplying both sides of this equation by 15 and then by 6 gives $6x + 6y = 15y$, from which we obtain $9y = 6x$ and then $y = \frac{2}{3}x$. The graph of this linear function is the line in Figure 13.

(b) The rate of change of y with respect to x is the slope $\frac{2}{3}$ of the line. \square

[†]Recall that two triangles are similar if they have the same angles.

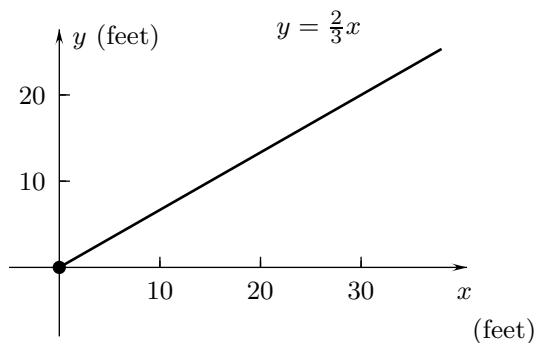


FIGURE 13

Approximating data with linear functions

Table 1 gives the results of a study of energy expenditure for men of different ages.⁽²⁾ The second row gives the average total daily energy expenditure for each age group; the third row gives the average daily basal energy expenditure (basal metabolism), which is the energy expended by the body apart from that used in activities; and the fourth row gives the average daily energy expenditure due to activity.

TABLE 1. ENERGY EXPENDITURES (CALORIES PER DAY)

Age	30	40	50	60	70	80
Total energy expenditure	2700	2650	2470	2360	2340	2110
Basal energy expenditure	1590	1600	1580	1560	1500	1400
Energy spent in activities	1110	1050	890	800	840	710

The data in Table 1 is plotted in Figure 14, where the upper string of dots gives the total energy expenditure, the dots in the middle give the basal energy expenditure, and the dots at the bottom give the energy expended in activities. Lines have been drawn in Figure 15 to approximate the dots.

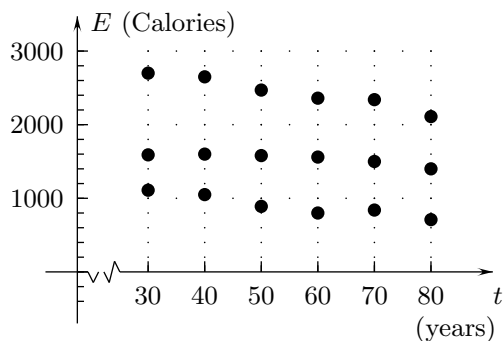


FIGURE 14

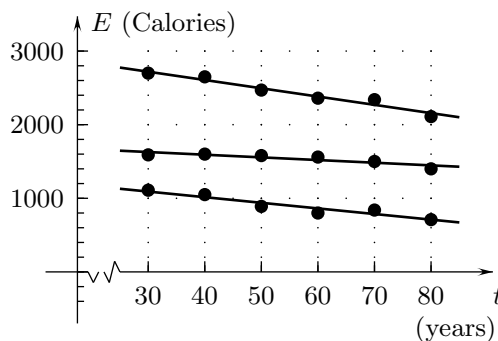


FIGURE 15

⁽²⁾Data adapted from “Nutrition and Aging” by C. Finch, *The Handbook of Aging*, New York: Van Nostrand Reinhold Company, Inc., 1985, p. 883.

Example 7 (a) Use approximate coordinates of two points on the upper line in Figure 15 to find an approximate equation for it. This gives a linear function $E = E_T(t)$ that is a model of total daily energy expenditure as a function of age. (b) Use an approximate equation of the middle line in Figure 15 to give a linear model $E = E_B(t)$ for the basal metabolism.

SOLUTION (a) The point at $t = 30$ on the top line is at $E \approx 2700$ and the point at $t = 80$ is at $E \approx 2200$. With these coordinates the slope would be $(2200 - 2700)/(80 - 30) = -10$ and the top line would have the equation $E = 2700 - 10(t - 30)$, which can be rewritten $E = 3000 - 10t$. With this equation for the line, we have $E_T(t) = 3000 - 10t$ (Calories). (Different approximations would give different formulas.)

(b) The point at $t = 30$ on the middle line is at $E \approx 1600$ and the point at $t = 80$ is at $E \approx 1400$. With these coordinates, the slope would be $(1400 - 1600)/(80 - 30) = -4$ and the middle line would have the equation $E = 1600 - 4(t - 30)$ or $E = 1720 - 4t$. This gives $E_B(t) = 1720 - 4t$ (Calories). \square

Example 8 Use the results of Example 6 to give a linear model for the energy $E = E_A(t)$ spent on activities.

SOLUTION The energy expended in activities is the total energy expenditure, minus the basal metabolism: $E_A(t) = E_T(t) - E_B(t) = (3000 - 10t) - (1720 - 4t) = 1280 - 6t$ Calories. \square

Interactive Examples 2.1

Interactive solutions are on the web page <http://www.math.ucsd.edu/~ashenk/>.[†]

1. The perimeter of a square of width w is $P = 4w$. What is the rate of change of the perimeter with respect to the width?
2. Give a formula for the linear function $y = p(w)$ whose constant rate of change is -3 and whose value at $w = 10$ is -6 .
3. Give a formula for a linear function $y = M(x)$ that approximates the data in the table below.

x	0	2	4	6	8
y	4.2	8.0	12.0	16.1	19.9

4. Give a formula for a linear function $y = L(x)$ that approximates the data in Figure 16.

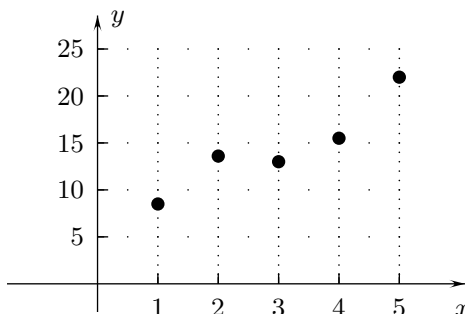


FIGURE 16

[†]In the published text the interactive solutions of these examples will be on an accompanying CD disk which can be run by any computer browser without using an internet connection.

5. The density of sucrose is 1.58 grams per milliliter.⁽³⁾ (a) Give a formula for the mass m (grams) of a sample of sucrose as a function of its volume V (milliliters). (b) What is the rate of change of the mass of a sample of sucrose with respect to its volume? Give the units in which this rate of change is measured.
6. A parachutist is 300 meters above the ground at time $t = 0$ seconds and descends at the constant rate of 15 meters per second until she reaches the ground. (a) Give a formula for her height $s = s(t)$ above the ground as a function of t . (b) When does she reach the ground? (c) Sketch the graph of $s = s(t)$.

Exercises 2.1

^AAnswer provided. ^OOutline of solution provided. ^CGraphing calculator or computer required.

CONCEPTS:

- How are the graphs of two linear functions related if they have the same constant rate of change with respect to their variable?
- What is the rate of change with respect to x of the linear function $y = F(x)$ if $F(x) = 10G(x)$ and the rate of change of the linear function $y = G(x)$ with respect to x is 5?
- What is the rate of change of a linear function $y = f(x)$ with respect to x if its graph is the line through the origin that is 45° above the positive x -axis and the scales on the axes are equal?
- How are the graphs of the linear functions $y = f(x)$ and $y = g(x)$ related if the constant rate of change of f with respect to x is 1, the constant rate of change of g with respect to x is -1 , and there are equal scales on the axes?
- What can you say about the rate of change of a linear function $y = f(x)$ with respect to x if $f(0) = 0$ and its graph passes through the acute angle AOB determined by the points $A(5, 2)$, $O(0, 0)$, and $B(2, 5)$?
- How are the rates of change of the revenue, cost, and profit related in Example 4?
- How is the rate of change of the volume of gasoline with respect to time in Example 5 related to the rate of change of the volume of gasoline with respect to distance and the rate of change of the distance with respect to time?

BASICS:

- ^O What is the constant rate of change of $y = 7 - 6x$ with respect to x ?
- ^A What is the constant rate of change of $f(x) = 27.4x + 6.54$ with respect to x ?
- What is the constant rate of change of $y = 470 - \frac{1}{3}z$ with respect to z ?
- ^O Give a formula for the linear function $y = f(x)$ whose constant rate of change with respect to x is 50 and whose value at $x = 3$ is 10.
- ^A A linear function $y = g(x)$ has the value 100 at $x = 0$ and its constant rate of change with respect to x is -40 . Give a formula for it.
- (a) Give a formula for the linear function $y = h(x)$ whose value at $x = 0.2$ is 6 and whose value at $x = 0.3$ is 2. (b) What are the rates of change of $y = h(x)$ and of $y = -h(x)$ with respect to x ?

⁽³⁾CRC Handbook of Chemistry and Physics, 62nd Edition, Boca Raton, FL: CRC Press, 1981, p. C-525.

- 14.⁰ Give a formula for the linear function $y = k(x)$ whose graph is shown in Figure 17.

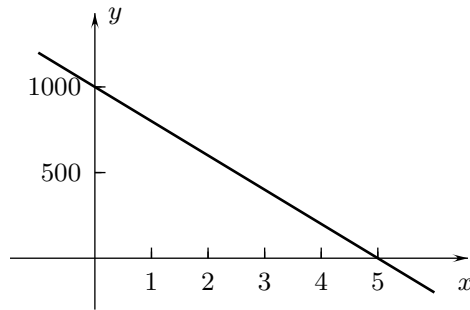


FIGURE 17

- 15.^A Give a formula for the linear function $y = q(x)$ whose graph is in Figure 18.

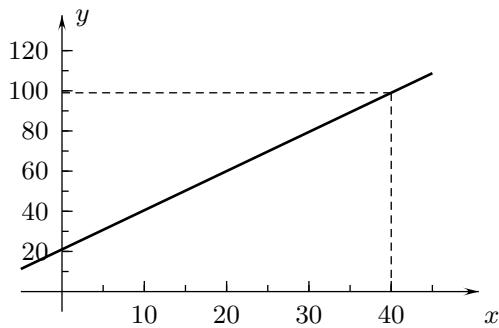


FIGURE 18

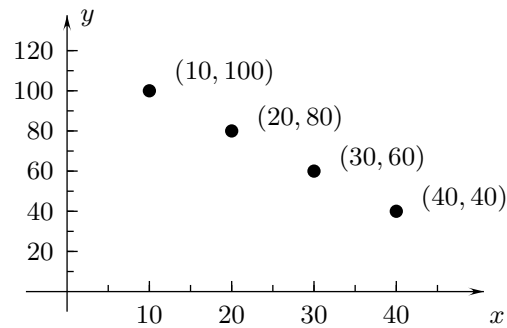


FIGURE 19

16. Find a formula for the linear function $y = L(x)$ whose graph contains the points in Figure 19.
 17.⁰ Give a formula for a linear function $y = L(x)$ that approximates the data in Figure 18.

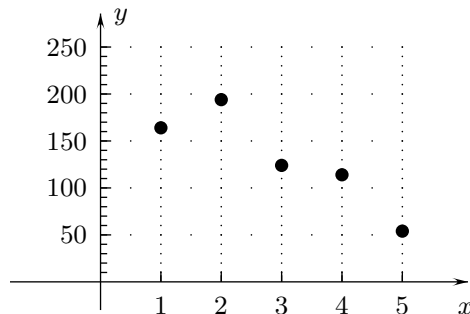


FIGURE 20

- 18.⁰ Give a formula for a linear function $y = M(x)$ that approximates the data in the table below.

x	0	5	10	15	20
y	25	124	226	323	525

19. (a) Give a formula for the linear function $y = K(x)$ that has the values at $x = 100$ and $x = 200$ in the next table. (b) What other data in the table does $y = K(x)$ fit?

x	100	200	300	400	500
y	50	65	80	96	109

20. What is the rate of change of the circumference of a circle with respect to its diameter?
- 21.^A The density of acetone is 49.4 pounds per cubic foot.⁽⁴⁾ (a) Give a formula for the weight w (pounds) of a sample of acetone as a function of its volume V (cubic feet). (b) What is the rate of change of the weight of a sample of acetone with respect to its volume?
22. The length L of a copper rod is a linear function of the temperature $T^\circ\text{F}$. The rod is 100 inches long at 50°F and expands 0.093 inches for every degree increase in its temperature.⁽⁵⁾ (a) Give a formula for L as a function of T . (b) At what temperature is the rod 100.5 inches long?
- 23.^A At the surface of the ocean, the water pressure equals the air pressure, which is 14.7 pounds per square inch. In one mathematical model, it is assumed that as you descend vertically into the ocean, the pressure increases at the constant rate of 0.44 pounds per square inch per foot.⁽⁶⁾ (a) Give a formula with this mathematical model for the water pressure p (pounds per square inch) as a function of the depth h (feet) beneath the surface. (b) Based on this model, what is the water pressure at the deepest spot in the oceans, Challenger Deep, which is 36,198 feet beneath the surface?
24. A hiker walks north at the constant speed of 3 miles per hour from 2:00 PM to 5:00 PM. At 2:00 PM he is five miles north of his house. Give his distance s from his house as a function of the number of hours t since 2:00 PM for $0 \leq t \leq 3$.
- 25.^A A balloon is 100 meters above the ground at time $t = 5$ seconds and rises at the constant rate of 10 meters per second. (a) Give a formula for its height $s = s(t)$ above the ground as a function of t . Sketch the graph of $s = s(t)$.
26. Figure 21 shows the graph of a linear function $O = O(V)$ that is a fairly good model of a human's oxygen absorption, measured in liters per minute, as a function of the rate of his or her breathing, also measured in liters per minute.⁽⁷⁾ Based on this mathematical model, what is the rate of change of oxygen absorption with respect to the rate of breathing?

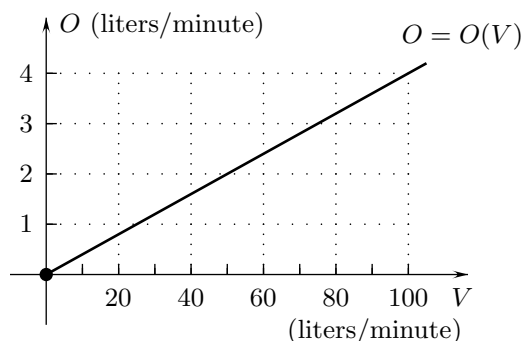


FIGURE 21

27. Which is greater, the rate of change of the circumference of a circle with respect to its diameter or the rate of change of the perimeter of a square with respect to its width?

⁽⁴⁾ *CRC Handbook of Chemistry and Physics*, 62nd Edition, Boca Raton, FL: CRC Press, 1981, p. F-3.

⁽⁵⁾ *Ibid*, p. D-161.

⁽⁶⁾ Data adapted from *The World Book Encyclopedia*, Vol. 14, Chicago IL: World Book, Inc., 1978, p.,491.

⁽⁷⁾ Data adapted from *Textbook of Medical Physiology* by A. Guynton, Philadelphia, PA: W. B. Saunders Co., 1976, p.

- 28.** The most commonly used scales for temperature—the Fahrenheit and Celsius scales—are related by linear functions.[†] The graph of degrees Fahrenheit T_F as a function of degrees Celsius T_C is the line in Figure 22. The inverse function in Figure 23 gives the temperature in degrees Celsius as a function of the temperature in degrees Fahrenheit. The dots on the graphs correspond to the freezing point of water, which is 0°C and 32°F , and to its boiling point, which is 100°C and 212°F . **(a)** Give formulas for T_F as a function of T_C and for T_C as a function of T_F . (Use fractions, not decimals.) **(b)** How are the rates of change of these functions related?

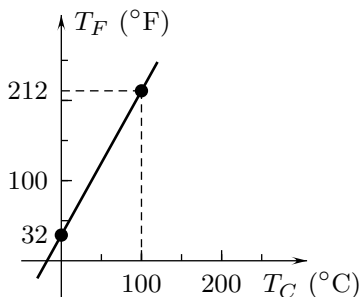


FIGURE 22

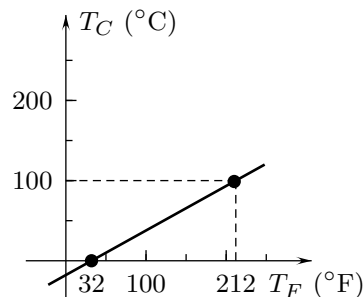


FIGURE 23

- 29.** A car rental agency charges \$25 per day plus 15 cents per mile. Let $F_1(x)$ be the charge for renting the car for one day and driving it x miles, let $F_2(x)$ be the charge for renting the car for two days and driving it x miles, and let $F_3(x)$ be the charge for renting the car for three days and driving it x miles. **(a)** Give formulas for $y = F_1(x)$, $y = F_2(x)$, and $y = F_3(x)$ and sketch their graphs in one xF -plane. **(b)** What are the rates of change of $y = F_1(x)$, $y = F_2(x)$, and $y = F_3(x)$ with respect to x ?
- 30.** The Kelvin scale for measuring temperature is calculated by adding a constant to the temperature measured in degrees Celsius, so that absolute zero, which is -273 degrees Celsius, is zero degrees Kelvin. **(a)** Give formulas for degrees Kelvin T_K , first as a function of degrees Celsius T_C and then as a function of degrees Fahrenheit T_F . ($T_C = \frac{5}{9}(T_F - 32)$ by Problem 20) **(b)** What is the rate of change of T_K with respect to T_F and how does it relate to the rate of change of T_C with respect to T_F ?

EXPLORATION:

- 31.** George and Martha are both driving south from Kansas City on Highway 71. George is $50 + 60t$ miles from Kansas City and Martha is $25 + 70t$ miles from Kansas City at time t (hours). **(a)** How fast is George driving? **(b)** How fast is Martha driving? **(c)** When and where does Martha catch up to George? **(d)** What is the rate of change of the distance between them before Martha passes George? **(e)** What is the rate of change of the distance between them after Martha passes George?
- 32.^A** A rectangular tank is 2 feet wide, 6 feet long, and 4 feet high. **(a)** What is the area of its base? **(b)** Give a formula for the volume $V(h)$ of water in the tank as a function of the depth of the water h , measured in feet. **(c)** What is the domain of the function V of part (b)? **(d)** What is the rate of change of V with respect to h ?

[†]The Fahrenheit scale was devised by Gabriel Daniel Fahrenheit (1686–1736), a German physicist who was the first to make mercury thermometers. The Celsius scale was introduced by a Swedish astronomer, Anders Celsius (1701–1744).

- 33.^A** The next table gives the annual world meat production and average meat production per capita in the years 1950, 1960, 1970, 1980, and 1990.⁽⁸⁾ **(a)** One of these sets of data can be modeled exactly by a linear function for 1950, 1960, 1970, and 1980. Find its formula. **(b)** What error and relative error are made if this model is used in 1990? (Recall that relative error equals the error divided by the correct value.)

ANNUAL WORLD MEAT PRODUCTION

Year = t	1950	1960	1970	1980	1990
Total production = x (million metric tons)	46	68	98	133	171
Per capita production = y (kilograms)	18	22	26	30	32

- 34.** **(a)** Plot the total meat production data in the table above as five points in a tx -plane. If this data were given by a linear function, the dots would lie on a line, but this is not the case. **(b)** Use the table to find slopes of the four lines joining (i) the dot at 1950 to the dot at 1960, (ii) the dot at 1960 to the dot at 1970, (iii) the dot at 1970 to the dot at 1980, and (iv) the dot at 1980 to the dot at 1990. **(c)** What is the average of the slopes from part (b)? **(d)** Show that the answer to part (c) equals the slope of the line joining the dot at 1950 to the dot at 1990.
- 35.** If you were buying a horse and wanted to have some idea of how much it would cost to feed it, you might consult a table as below, which lists typical daily feed requirements of horses of various weights if they are ridden five hours per day. The graph of this data is shown in Figure 24.⁽⁹⁾ Find a linear function that approximates the data in the table and draw it with the dots in Figure 23.

DAILY FEED REQUIREMENTS OF HORSES

Weight w (pounds)	800	900	1000	1100	1200
Feed requirement F (pounds)	18.7	20.5	22.2	23.8	25.4

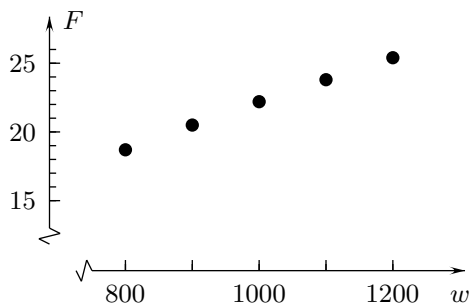


FIGURE 24

⁽⁸⁾Data adapted from *Vital Signs, 1992* by Brown, Flavin, and Kane, New York, NY: W.W. Norton & Company, Inc., 1992, p. 29.

⁽⁹⁾Data adapted from from *The Science of Animal Husbandry*, by J. Blakey, Reston, VA: Reston Publishing Company, 1974, p. 343.

- 36.^A** Federal income tax in 2001 on taxable income between \$28,400 and \$68,800 for a single person was \$3,910 plus 25% of the amount over \$28,400.⁽¹⁰⁾ **(a)** Give a formula for the tax T on taxable income x for $28,400 \leq x \leq 65,800$. **(b)** Imagine that up to August 1, 2003 you had earned 2003 taxable income of \$30,000, and that your taxable income increased at the constant rate of \$100 per day through the month. Give a formula for your taxable income x as a function of the time t (days) after the beginning of the month for $0 \leq t \leq 31$. **(c)** Give a formula for your federal tax as a function of t for $0 \leq t \leq 31$. **(d)** What are the rates of change of T with respect to x , of x with respect to t , and of T with respect to t , and how are these related?
- 37.^A** A crate is being hauled at the constant rate of 5 feet per minute up a ramp that has slope $\frac{3}{4}$. Let s denote the distance the crate has traveled up the ramp, let x be the corresponding change in the crate's horizontal coordinate, and let y be the change in its vertical coordinate (Figure 25), all measured in feet. **(a)** Find a formula for s as a function of the amount of time t (minutes) the crate has been pulled up the ramp, starting at the bottom of the ramp. **(b)** Find formulas for x and y as functions of s . **(c)** Give formulas for x and y as functions of t . **(d)** What are the rates of change of x and y with respect to t ?

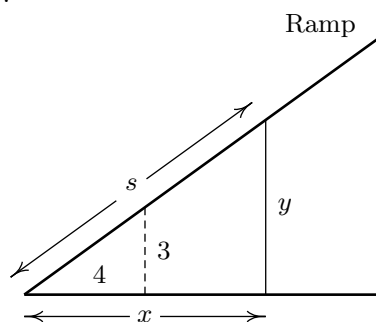


FIGURE 25

(End of Section 2.1)

⁽¹⁰⁾Data from *1040 Forms and Instructions*, Bloomington, IL: Department of the Treasury, Internal Revenue Service, 2003, p. 74.