Section 2.R

(3/1908)

Review exercises

1.[A] What is the constant rate of change of $f(x) = 10x + 6$ with respect to x?

2.[A] Give a formula for the linear function $y = g(x)$ whose constant rate of change with respect to x is -3 and whose value at $x = -4$ is 5.

3.[A] A linear function $y = P(x)$ has the value 10 at $x = 5$ and its constant rate of change is 6. Give a formula for it.

4.[A] Give a formula for the linear function $y = R(x)$ whose graph contains the points in Figure 1.

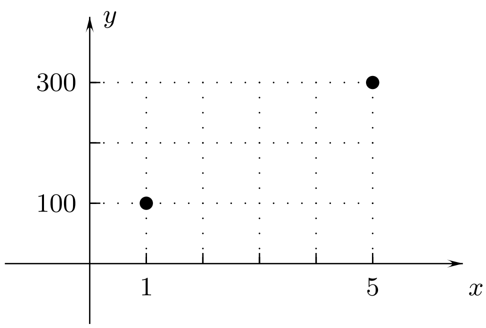

FIGURE 1

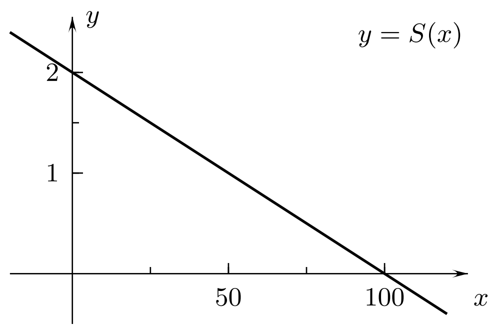

FIGURE 2

5.[A] Give a formula for the linear function $y = S(x)$ whose graph is the line in Figure 2.

6.[A] **(a)** Find the exact and approximate decimal values of the average rate of change of $T(x) = 3 + 3x - e^x$ from $x = -1$ to $x = 2$. [C]**(b)** Generate in the window $-2 \le x \le 3, -2 \le y \le 5$ the graph of T and the secant line whose slope is the average rate of change from part (a).

7.[A] **(a)** Find the exact and approximate decimal values of the average rate of change of $y = \tan x$ from $x = -1.3$ to $x = 1.3$. [C]**(b)** Generate in the window $-\frac{1}{2}\pi \le x \le \frac{1}{2}\pi, -7.5 \le y \le 7.5$ the graph of the function and the secant line whose slope is the average rate of change from part (a).

8.[A] **(a)** Find the approximate average rate of change from $x = -3$ to $x = 3$ of the function $y = f(x)$ of Figure 3. **(a)** Draw the secant line whose slope is the average rate of change from part (a).

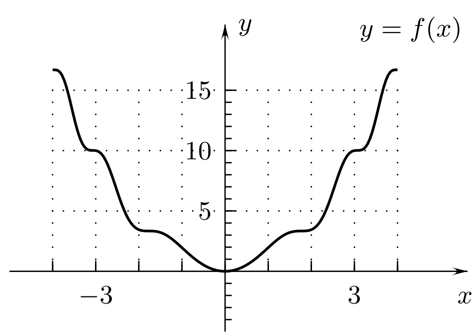

FIGURE 3

9.[A] **(a)** Find the approximate average rate of change from $x = 1$ to $x = 3$ of the function $y = g(x)$ of Figure 4. **(b)** Draw the secant line whose slope is the average rate of change from part (a).

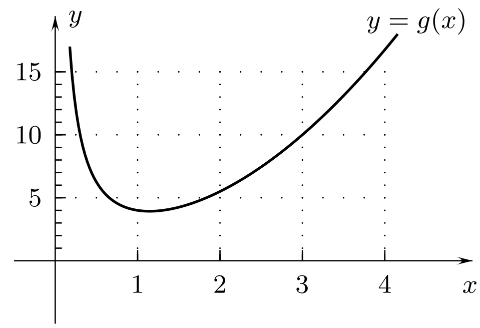

FIGURE 4

10.[A] The U. S. trade deficit was 108 billion dollars per year in June, 1995, 420 billion dollars per year in June, 2000, and 700 billion dollars per year in June, 2005.[1] What were the average rates of change of the trade deficit **(a)** from June, 1995, to June, 2000, and **(b)** from June, 2000, to June, 2005?

11.[A] Calculate the slope of the secant line through the points at $x = a$ and $x = b$ on the graph of $P(x) = \dfrac{25x}{x^2 + 1}$ with $a = 2$ and $b = 4, 3, 2.1, 2.001$ and 2.00001. Use the results to predict the value of the derivative $P'(2)$. [C] If you use the secant-line program, set the window to $-1 \le x \le 6, -2 \le y \le 15$.

12.[A] Calculate the slope of the secant line through the points at $x = a$ and $x = b$ on the graph of $Q(x) = 25 - x^4$ with $a = -1$ and $b = 1, 0, -0.9, -0.999$ and $= 0.99999$. Use the results to predict the value of the derivative $Q'(-1)$. [C] If you use the secant-line program, set the window to $-2.5 \le x \le 2.5, -5 \le y \le 30$.

Use the definition $f'(a) = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ to find the derivatives in Exercises 13 and 14.

13.[A] $R'(2)$ for $R(x) = x^2 - 2x + 1$

14.[A] $S'(3)$ for $S(x) = 3 + 3/x$

15.[A] Give an equation of the tangent line at $x = 2$ to the graph of the function $R(x) = x^2 - 2x + 1$ from Exercise 13.

16.[A] Give an equation of the tangent line at $x = 3$ to the graph of the function $s(x) = 3 + 3/x$ from Exercise 14.

In Exercises 17 and 18 use the Δx-formulation of the definition to find the derivatives.

17.[A] $f'(5)$ for $f(x) = 25 - x^2$

18.[A] $g'(x)$ for $g(x) = 4/x$

[1] Data adapted from www.economics. about.com and the *San Diego Union-Tribune*, August 28, 2005.

19.[A] Figure 5 shows the graph of a balloon's height $h = h(t)$ (meters) above the ground as a function of the time t (minutes), the tangent line to the graph at $t = 8$, and two points on the tangent line. What is the rate of change of the height with respect to time at $t = 8$? (Give the units.)

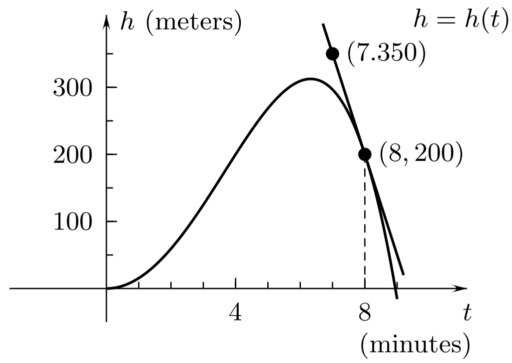

FIGURE 5

Find the derivatives in Exercises 20 through 23.

20.[A] $\dfrac{d}{dx}(5x^{-1} - 6x^{-3} - 5^{-4})$

21.[A] $\dfrac{d}{dx}(\sqrt[4]{x} - \sqrt[6]{x})$

22.[A] $\dfrac{d}{dt}\Big[2t^3 - 3t^2\Big]_{t=10}$

23.[A] $\left[\dfrac{d}{dt}\left(\dfrac{2}{\sqrt{t}} - \sqrt{t}\right)\right]_{t=1}$

In Exercises 24 and 25 **(a)** find exact equations of the tangent lines at the given values of x. [C]**(b)** Generate the curves and the tangent lines together in the indicated windows, and copy the drawings on your paper.

24.[A] The tangent line to $y = 2 + 2x^2 + x^{-1}$ at $x = 1$ $(0 \le x \le 3, 0 \le y \le 15)$

25.[A] The tangent line to $y = 10 - 8x^{-1/2}$ at $x = 4$ $(-1 \le x \le 8, -6 \le y \le 10)$

In Exercises 26 and 27, find the linear approximation L of the function f at $x = a$.

26.[A] $f(x) = x^{2/3}$; $a = 8$

27.[A] $f(x) = \dfrac{1}{x} - \dfrac{2}{x^2}$; $a = 2$

In Exercises 28 and 29, give an equation relating df and dx at $x = a$.

28.[A] $f(x) = x + x^2 + x^3$; $a = 2$

29.[A] $f(x) = 3 + 4\sqrt[4]{x^3}$; $a = 1$

30.[A] What is the approximate value of $Z = W(y)$ at $y = 4.999$ if $W(5) = 10$ and $W'(5) = -6$?

31.[A] At the beginning of October, 2005, the interest rate for 30-year fixed-rate home mortgages was 6 percent and was increasing 3.6 percent per year.[(2)] What was the approximate rate at the beginning of November, 2005?

32.[A] The volume of a cube is measured to be 1000 cubic inches with an an error ≤ 0.6 cubic inches. **(a)** What is the width of the cube if its volume is exactly 1000 cubic inches? **(b)** Use differentials to estimate the maximum error in the measured width of the cube.

33.[A] Sketch the graph of the function $y = F(x)$ and of its derivative $r = F'(x)$, where

$$F(x) = \begin{cases} x^2 & \text{for} \quad -2 \le x < 2 \\ 8 - 2x & \text{for} \quad x \ge 2. \end{cases}$$

[(2)]Data adapted from *San Diego Union Tribune*, January 1, 2006, Source: Freddie Mac.

34.[A] Figure 6 shows the graph of the median price $P = P(t)$ of a home in San Diego as a function of the time t (months) with $t = 0$ on December 1, 2004.[(3)] What was the approximate rate of change of the median price on May 1 (at $t = 5$)?

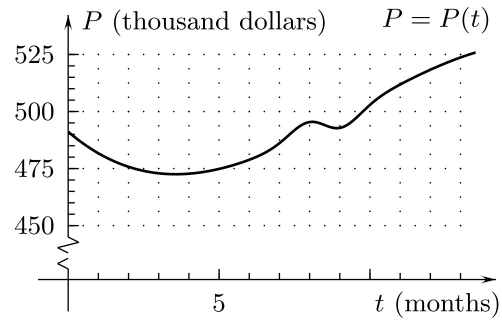

FIGURE 6

35.[A] The table below gives the average of personal savings as percentage of disposal income in the United States in various years.[(4)] Based on this information what was the approximate rate of decrease of the savings percentage **(a)** in 1981 and **(b)** in 2000?

t (year)	1980	1985	1990	1995	2000	2005
$P(t)$ (%)	8.8	8.1	7.1	4.2	2.3	−1.8

36.[A] What is the third derivative of $y = 3x - 2x^4 + x^7$?

37.[A] What is the acceleration of an object whose coordinate on an s-axis at time t (hours) is $s = t^{5/2}$ (meters)?

Find the derivatives in Exercises 38 through 43. Do not simplify the answers.

38.[A] $\frac{d}{dx}[(x^4 + 6x - 5)(x^5 + x^4)]$

39.[A] $\frac{d}{dx}[(x^{-1} + x^{-2})(x + x^2)]$

40.[A] $\frac{d}{dx}\left(\frac{1 + x^5}{x - x^3}\right)$

41.[A] $\frac{d}{dx}\left(\frac{x - 3}{x^2 + 5x - 1}\right)$

42.[A] $h'(5)$ where $h(t) = f(t)g(t)$, $f(5) = 100, g(5) = 2, f'(5) = -7$ and $g'(5) = 4$

43.[A] $T'(100)$ where $T(z) = \frac{R(z)}{S(z)}, R(100) = 8, S(100) = 2, R'(100) = -5$, and $S'(100) = 1$

Find the derivatives in Exercises 44 through 47.

44.[A] $\frac{d}{dx}[(x^2 - 3x)^7]$

45.[A] $\frac{d}{dx}\left[\frac{1}{\sqrt{5x - 4}}\right]$

46.[A] $Q'(3)$ where $Q(x) = [R(x)]^3, R(3) = 10$, and $R'(3) = -5$

47.[A] $f'(x)$ where $f(x) = (ax^3 + bx + c)^4$ with constants a, b, and c

48.[A] **(a)** Give an equation of the tangent line to $y = \sqrt{1 - x}$ at $x = 0$. [C]**(b)** Generate the graph of the function and the tangent line in the window $-3 \le x \le 2, -0.5 \le y \le 3$.

[(3)]Data adapted from *The San Diego Union Tribune*, January 5, 2006, Source: DataQuick Information Systems.
[(4)]Data adapted from *The Wall Street Journal*, January 1, 2006., Soirce: U. S. Bureau of Economic Analysis.

49. Figure 7 shows the graph of the area $A = A(t)$ (square meters) of a square as a function of the time t (minutes). At what approximate rate is the width of the square increasing or decreasing at $t = 3$?

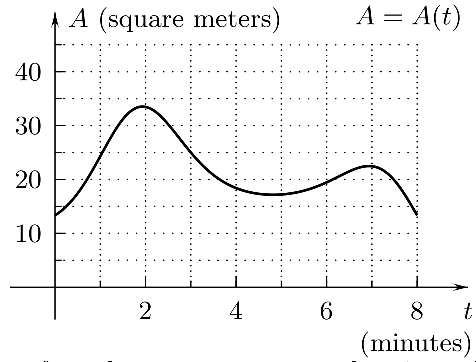

FIGURE 7

50.[A] What is the rate of change of the volume of a sphere at a moment when its radius is 2 inches and is increasing 10 inches per hour?

(End of Section 2.R)