

Section 6.R

(3/20/08)

Review exercises

1.^A A car's velocity on a straight road at time t (hours) is 60 miles per hour for $1 < t < 4$ and 75 miles per hour for $4 < t < 6$. How far does it travel for $1 \leq t \leq 6$?

2.^A What is $f(5)$ if $y = f(x)$ is continuous on $[-5, 5]$, $f(-5) = -10$, and $r = f'(x)$ is a step function with $f'(x) = 4$ for $-5 < x < 0$, and $f'(x) = 6$ for $0 < x < 5$?

Express the sums in Exercises 3 and 4 with summation notation.

3.^A $1 + \frac{1}{10} + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^3 + \left(\frac{1}{10}\right)^4 + \left(\frac{1}{10}\right)^5 + \left(\frac{1}{10}\right)^6$

4.^A $x^5 + x^6 + x^7 + x^8 + \cdots + x^{100}$

5.^A Use the formula for the area of a trapezoid to find the value of $\int_0^5 (2x + 5) dx$.

6.^A Use the formula for the area of a circle to evaluate $\int_{-3}^3 \sqrt{9 - x^2} dx$.

7.^A Use areas to evaluate $\int_0^{20} g(x) dx$ where $g(x) = \begin{cases} x & \text{for } 0 < x < 10 \\ 10 & \text{for } 10 < x < 20. \end{cases}$

8.^A Calculate directly, without using Riemann sum program, the right Riemann sum for $\int_0^1 \sqrt{x} dx$ corresponding to the partition $0 < \frac{1}{4} < \frac{1}{2} < \frac{3}{4} < 1$.

c 9.^A Use a Riemann sum program to calculate the right Riemann sum for $\int_{-2}^2 (5 - x^2) dx$ corresponding to the partition of $[-2, 2]$ into six equal subintervals. Draw the graph of the function and the rectangles whose areas give the Riemann sum.

c 10.^A Use a Riemann sum program to calculate the left Riemann sum for $\int_0^\pi (2 - 3 \sin x) dx$ corresponding to the partition of $[0, \pi]$ into five equal subintervals. Draw the graph of the function and the rectangles whose areas give the Riemann sum.

11.^A What is $\int_0^{70} [5f(x) - 10g(x)] dx$ if $\int_0^{70} f(x) dx = 12$ and $\int_0^{70} g(x) dx = 4$?

12.^A What is the value of $\int_0^3 \frac{d}{dx} (\sqrt{x^2 + 16}) dx$?

13.^A Evaluate $\int_0^{12} \frac{d}{dx} \sqrt{x^2 + 25} dx$.

14.^A What is $F(10)$ if F is continuous and F' is piecewise continuous on $[0, 10]$, $\int_0^{10} F'(x) dx = 100$, and $F(0) = -50$?

Perform the integration in Exercises 15 through 20.

15.^A $\int (2x^{-3} - 3x^{-2}) dx$

18.^A $\int 10x^{1/10} dx$

16.^A $\int \sqrt{x} dx$

19.^A $\int_{1000}^{1002} 3 dx$

17.^A $\int_0^{10} (x^3 - x^4) dx$

20.^A $\int_0^1 (1 + x + x^2) dx$

21.^A Find the value of $\int_{-1}^2 G(x) dx$ where $G(x) = \begin{cases} 5x^4 & \text{for } x < 1 \\ 10 & \text{for } x > 1. \end{cases}$

In Exercises 22 through 25 Find the areas of the given regions.

22.^A The region between $y = 1 - x^4$ and the x -axis.

23.^A The region between $y = 1 + x^4$ and the x -axis for $-1 \leq x \leq 1$.

24.^A The region between $y = -1 - x^2$ and the x -axis for $-1 \leq x \leq 0$.

25.^A The region between $y = 1 + 2x + 3x^2 + 4x^3$ and the x -axis for $0 \leq x \leq 2$.

26.^A A tank that leaks one gallon of water per minute contains 8 gallons of water at $t = 0$ (minutes). Water is added at the rate of $10t$ gallons per minute for $0 \leq t \leq 2$. How much water is in the tank at $t = 2$?

27.^A The temperature outside a house is 10° F at $t = 0$ and is rising $10 + \frac{1}{4}t$ degrees per hour at time t or three hours. What is the temperature at time t for $0 \leq t \leq 4$?

28.^A A reservoir contains 300 acre-feet of water at $t = 1$ (days) and water is drained off at the rate of $100/t^2$ acre-feet per day for four days. How much water is in the reservoir at $t = 5$?

Find the derivatives in Exercises 29 through 32.

29.^A $\frac{d}{dx} \int_0^x \sin^2 t dt$

31.^A $\frac{d}{dx} \int_{x^2}^{x^3} \sqrt{1+t} dt$

30.^A $\frac{d}{dx} \int_x^0 e^{3t} dt$

32.^A $\frac{d}{dx} \int_{\cos x}^{\sin x} \frac{1}{t+5} dt$

Carry out the integration in Exercises 33 through 50.

33.^A $\int \left(\frac{1}{x} + 2e^x + 3 \sin x + 4 \cos x \right) dx$

42.^A $\int_1^5 \sqrt{2x+1} dx$

34.^A $\int_1^{10} \left(\frac{3}{x} - 5 \sin x \right) dx$

43.^A $\int (4x+1)^{1/3} dx$

35.^A $\int (\sec^2 x + \csc^2 x) dx$

44.^A $\int_0^2 \frac{x^2}{\sqrt{x^3+1}} dx$

36.^A $\int_{-3}^3 (x + e^x) dx$

44.^A $\int \frac{x}{(x^2+3)^2} dx$

37.^A $\int (5^x + 6^x) dx$

45.^A $\int e^x(1+e^x)^2 dx$

38.^A $\int_0^1 x\sqrt{x^2+9} dx$

46.^A $\int_0^\pi \frac{\sin x}{(2+\cos x)^2} dx$

39.^A $\int_0^1 \frac{x^2}{6+x^3} dx$

49.^A $\int_0^{\pi/4} \tan^2 x \sec^2 x dx$

40.^A $\int \frac{e^y}{1+e^y} dy$

50.^A $\int_0^2 \frac{e^x}{\sqrt{8+e^x}} dx$

41.^A $\int \frac{1+\cos x}{x+\sin x} dx$

51.^A $\int_0^{\pi/2} \cos x(1+\sin x)^3 dx$

Find the areas of the regions in Exercises 52 through 54.

52.^A The region bounded by the curve $y = \sqrt{9 - x}$ and by the x - and y -axes

53.^A The region between $y = x\sqrt{9 - x^2}$ and the x -axis for $0 \leq x \leq 3$

54.^A The region between $y = \frac{e^x}{1 + e^x}$ and the x -axis for $0 \leq x \leq 1$

(End of Section 6.R)