Section 6.R
Review exercises

1. A car’s velocity on a straight road at time $t$ (hours) is 60 miles per hour for $1 < t < 4$ and 75 miles per hour for $4 < t < 6$. How far does it travel for $1 \leq t \leq 6$?

2. What is $f(5)$ if $y = f(x)$ is continuous on $[-5, 5]$, $f(-5) = -10$, and $r = f'(x)$ is a step function with $f'(x) = 4$ for $-5 < x < 0$, and $f'(x) = 6$ for $0 < x < 5$?

Express the sums in Exercises 3 and 4 with summation notation.

3. $1 + \frac{1}{10} + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^3 + \left(\frac{1}{10}\right)^4 + \left(\frac{1}{10}\right)^5 + \left(\frac{1}{10}\right)^6$

4. $x^5 + x^6 + x^7 + x^8 + \cdots + x^{100}$

5. Use the formula for the area of a trapezoid to find the value of $\int_5^0 (2x^2 + 5) \, dx$.

6. Use the formula for the area of a circle to evaluate $\int_{-3}^{20} \sqrt{9 - x^2} \, dx$.

7. Use areas to evaluate $\int_0^{20} g(x) \, dx$ where $g(x) = \begin{cases} x & \text{for } 0 < x < 10 \\ 10 & \text{for } 10 < x < 20. \end{cases}$

8. Calculate directly, without using Riemann sum program, the right Riemann sum for $\int_0^1 \sqrt{x} \, dx$ corresponding to the partition $0 < \frac{1}{4} < \frac{1}{2} < \frac{3}{4} < 1$.

9. Use a Riemann sum program to calculate the right Riemann sum for $\int_{-2}^{2} (5 - x^2) \, dx$ corresponding to the partition of $[-2, 2]$ into six equal subintervals. Draw the graph of the function and the rectangles whose areas give the Riemann sum.

10. Use a Riemann sum program to calculate the left Riemann sum for $\int_0^\pi (2 - 3 \sin x) \, dx$ corresponding to the partition of $[0, \pi]$ into five equal subintervals. Draw the graph of the function and the rectangles whose areas give the Riemann sum.

11. What is $\int_0^{70} [5f(x) - 10g(x)] \, dx$ if $\int_0^{70} f(x) \, dx = 12$ and $\int_0^{70} g(x) \, dx = 4$?

12. What is the value of $\int_0^3 \frac{d}{dx} \left(\sqrt{x^2 + 16}\right) \, dx$?

13. Evaluate $\int_0^{12} \frac{d}{dx} \sqrt{x^2 + 25} \, dx$.

14. What is $F(10)$ if $F$ is continuous and $F'$ is piecewise continuous on $[0, 10]$, $\int_0^{10} F'(x) \, dx = 100$, and $F(0) = -50$?

Perform the integration in Exercises 15 through 20.

15. $\int (2x^{-3} - 3x^{-2}) \, dx$  
16. $\int \sqrt{x} \, dx$  
17. $\int_0^{10} (x^3 - x^4) \, dx$  
18. $\int 10x^{1/10} \, dx$  
19. $\int_{1002}^{1000} 3 \, dx$  
20. $\int_0^1 (1 + x + x^2) \, dx$
21. Find the value of $\int_{-1}^{2} G(x) \, dx$ where $G(x) = \begin{cases} 5x^4 & \text{for } x < 1 \\ 10 & \text{for } x > 1. \end{cases}$

In Exercises 22 through 25 Find the areas of the given regions.

22. The region between $y = 1 - x^4$ and the $x$-axis.
23. The region between $y = 1 + x^4$ and the $x$-axis for $-1 \leq x \leq 1$.
24. The region between $y = -1 - x^2$ and the $x$-axis for $-1 \leq x \leq 0$.
25. A tank that leaks one gallon of water per minute contains 8 gallons of water at $t = 0$ (minutes). Water is added at the rate of $10t$ gallons per minute for $0 \leq t \leq 2$. How much water is in the tank at $t = 2$?

26. A The region between $y = 1 - x^4$ and the $x$-axis.
27. A The region between $y = 1 + x^4$ and the $x$-axis for $-1 \leq x \leq 1$.
28. A The region between $y = -1 - x^2$ and the $x$-axis for $-1 \leq x \leq 0$.
29. A The region between $y = 1 + 2x + 3x^2 + 4x^3$ and the $x$-axis for $0 \leq x \leq 2$.

Find the derivatives in Exercises 29 through 32.

29. $\frac{d}{dx} \int_{0}^{x} \sin^2 t \, dt$
30. $\frac{d}{dx} \int_{x}^{0} e^{3t} \, dt$
31. $\frac{d}{dx} \int_{0}^{x} \frac{x^3}{\sqrt{1 + t}} \, dt$
32. $\frac{d}{dx} \int_{\cos x}^{\sin x} \frac{1}{t + 5} \, dt$

Carry out the integration in Exercises 33 through 50.

33. $\int \left( \frac{1}{x} + 2e^x + 3 \sin x + 4 \cos x \right) \, dx$
34. $\int_{1}^{10} \left( \frac{3}{x} - 5 \sin x \right) \, dx$
35. $\int (\sec^2 x + \csc^2 x) \, dx$
36. $\int_{-3}^{3} (x + e^x) \, dx$
37. $\int (5^x + 6^x) \, dx$
38. $\int_{0}^{1} x \sqrt{x^2 + 9} \, dx$
39. $\int_{0}^{1} \frac{x^2}{6 + x^2} \, dx$
40. $\int \frac{e^y}{1 + e^y} \, dy$
41. $\int_{-3}^{1} \frac{1 + \cos x}{x + \sin x} \, dx$
42. $\int_{1}^{5} \sqrt{2x + 1} \, dx$
43. $\int (4x + 1)^{1/3} \, dx$
44. $\int_{0}^{2} \frac{x^2}{\sqrt{x^3 + 1}} \, dx$
45. $\int e^x (1 + e^x)^2 \, dx$
46. $\int_{0}^{\pi} \frac{\sin x}{(2 + \cos x)^2} \, dx$
47. $\int_{0}^{\pi/4} \tan^2 x \sec^2 x \, dx$
48. $\int_{0}^{\pi/2} \frac{\tan^2 x}{1 + \tan^2 x} \, dx$
49. $\int_{0}^{\pi/2} \frac{e^x}{\sqrt{8 + e^x}} \, dx$
50. $\int_{0}^{\pi/2} \frac{e^x}{\cos x(1 + \sin x)^3} \, dx$
51. $\int_{0}^{\pi/2} \frac{e^x}{\cos x(1 + \sin x)^3} \, dx$
Find the areas of the regions in Exercises 52 through 54.

52. The region bounded by the curve \( y = \sqrt{9 - x} \) and by the \( x \)- and \( y \)-axes

53. The region between \( y = x\sqrt{9 - x^2} \) and the \( x \)-axis for \( 0 \leq x \leq 3 \)

54. The region between \( y = \frac{e^x}{1 + e^x} \) and the \( x \)-axis for \( 0 \leq x \leq 1 \)

(End of Section 6.R)