CALCULUS
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(The page numbers are for sample Chapters 1, 2, 6, and 14.)

Introduction .................................................................p. xi
To the instructor ...........................................................p. xiii

Chapter 0. Mathematical Models: Functions and Graphs
0.1 Measuring and modeling: variables and functions
Variables, functions, and graphs. Functions given by formulas, tables, and graphs. Finding formulas for mathematical models. When is a curve the graph of a function? Change, percent change, and relative change.

0.2 Set notation and solving inequalities
Intervals and other sets of numbers. The absolute value function. Working with inequalities.

0.3 Power and exponential functions

0.4 Inverse functions and logarithms

0.5 Trigonometric and inverse trigonometric functions

0.6 Linear combinations, products, quotients, and compositions
Linear combinations, products, and quotients of functions. Polynomial and rational functions. Composite functions.

0.R Review Exercises

Chapter 1. Limits and Continuity
1.1 Finite limits ............................................................p. 1

1.2 More on finite limits ..................................................p. 14
Finding limits of quotients when the numerators and denominators tend to zero. Rationalizing differences of square roots

1.3 Continuity ...............................................................p. 20

1.4 Limits involving infinity .............................................p. 36
Infinite limits as $x \rightarrow \pm \infty$. Finite limits as $x \rightarrow \pm \infty$. One-sided and two-sided infinite limits. Infinite limits of transcendental functions.
1.5 **Formal definitions of limits** ................................................................. p. 51

1.R **Review Exercises** ................................................................. p. 62

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**Chapter 2. The Derivative and Applications**

2.1 **Linear functions and constant rates of change** ............................ p. 65
    Constant velocity. Other constant rates of change. Approximating data with linear functions.

2.2 **Average rates of change** .......................................................... p. 78
    Average velocity. Other average rates of change. Estimating instantaneous velocity with average velocity.

2.3 **Tangent lines, rates of change, and derivatives** .............................. p. 89
    Tangent lines, derivatives, and instantaneous rates of change. Predicting a derivative by calculating difference quotients. Finding exact derivatives. Equations of tangent lines. The $\Delta x\Delta f$-formulation of the definition of the derivative. Finding a rate of change from a tangent line. A secant line program.

2.4 **Derivatives of power functions and linear combinations** ................ p. 107
    Leibniz notation and the differentiation operator. The derivative of $y = x^n$. Derivatives of linear combinations of functions.

2.5 **Derivatives as functions and estimating derivatives** ....................... p. 118

2.6 **Derivatives of products and quotients** ......................................... p. 137
    The Product Rule. Related-rate problems. The Quotient Rule. The derivative of $y = x^n$ for integers $n \geq 2$ and Mathematical Induction.

2.7 **Derivatives of powers of functions** ............................................. p. 149
    The Chain Rule for powers of functions. On the order of operations.

2.8 **Linear approximations and differentials** ..................................... p. 157

2.R **Review exercises** ................................................................. p. 166
Chapter 3. Derivatives of Transcendental Functions

3.1 The general Chain Rule
An example with linear functions. The general Chain Rule. The Chain Rule in narrative problems. Using the Chain Rule with graphs.

3.2 Derivatives of logarithms
The derivative of \( y = \log_b x \) and the number \( e \). Derivative of the natural logarithms. Derivatives of natural logarithms of functions.

3.3 Derivatives of exponential functions
Growth factors. Modeling with exponential functions. Radioactive decay and half-life. Compound interest. Interest compounded continuously. The derivatives of \( y = e^x \), \( y = e^{u(x)} \), and \( y = b^{u(x)} \). The derivative of \( y = x^n \) for irrational \( n \).

3.4 The differential equation \( \frac{dy}{dt} = ry \)
Relative rates of change. The differential equation \( \frac{dy}{dt} = ry \) with constant \( r \). Newton’s Law of heating and cooling.

3.5 Derivatives of the sine and cosine functions
Derivatives of \( y = \sin x \) and \( y = \cos x \). The Chain Rule for sines and cosines of functions. Sinusoidal functions.

3.6 Derivatives of the other trigonometric functions
Derivatives of the tangent, cotangent, secant, and cosecant functions. The Chain Rule for the tangent, cotangent, secant, and cosecant functions. Derivatives of the inverse sine and cosine functions. The Chain Rule for the inverse sine and cosine functions. Derivatives of the inverse tangent and cotangent functions. The Chain Rule for the inverse tangent and cotangent functions.

3.7 The hyperbolic functions and their derivatives
The hyperbolic sine and cosine functions and their graphs. Hyperbolic functions and hyperbolas. The other hyperbolic functions. Inverse hyperbolic functions.

3.8 Review exercises

Chapter 4. Graphing and Optimization

4.1 First-Derivative Tests
Local maxima and minima: a necessary condition. The Mean Value Theorem. The First-Derivative Test for increasing and decreasing functions. Local maxima and minima: a sufficient condition.

4.2 Sketching graphs of rational functions
Sketching graphs of polynomials and other rational functions using First-Derivative Tests.

4.3 Second-Derivative Tests

4.4 Sketching graphs of nonrational functions
Functions constructed from powers with restricted domains. Sketching graphs of nonrational functions.

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†Formulas for derivatives of logarithms are derived in Section 3.2 using (i) algebraic properties of \( y = \log_b x \), (ii) the continuity of \( y = \log_b x \) and (iii) the existence of the limit \( e = \lim_{t \to \infty} (1 + 1/t)^t \). Facts (ii) and (iii) are established in Section 6.4, where the definition of \( y = \ln x \) as an integral is discussed.
4.5 Maxima and minima on intervals
The Extreme Value Theorem. Maxima and minima on finite closed intervals. Maxima and minima on other intervals.

4.6 Applied maximum/minimum problems
Finding extreme values with direct use of geometric formulas. Using formulas obtained from rates of change. Problems with constraints.

4.R Review exercises

Chapter 5. Further Applications of Derivatives

5.1 L’Hopital’s Rule
Indeterminate forms of types 0/0. Predicting limits with tangent line approximations. The Generalized Mean Value Theorem. L’Hopital’s Rule for limits of indeterminate forms of types 0/0, ∞/∞, 0 ∙ ∞, 10, 1∞, ∞0, and ∞ − ∞.

5.2 More related rate problems
Related rate problems using formulas derived from similar and equilateral triangles, the Pythagorean Theorem and other techniques.

5.3 Implicit differentiation
Implicitly defined functions. Derivatives of implicitly defined functions. Tangent lines to curves given by equations in x and y. Derivatives of inverse functions.

5.4 Newton’s method
Tangent line approximations and Newton’s method. How Newton’s method can fail.

5.R Review exercises

Chapter 6. The Integral and Applications

6.1 Step function rates of change ......................................................... p. 171
Step function rates of change. Approximating continuous rates of change with step functions.

6.2 The definite integral ................................................................. p. 180

6.3 The Fundamental Theorem, Part I ............................................. p. 198
Another look at Section 6.1. The Fundamental Theorem, Part I

6.4 The Fundamental Theorem, Part II ........................................... p. 209
The Fundamental Theorem, Part II. Another proof of Part I of the Fundamental Theorem. Derivatives of integrals with functions as limits of integration. Defining \( y = \ln x \) as an integral.

6.5 Integrals of powers and indefinite integrals ..................................... p. 218
Integrals of \( y = x^n \) with \( n \neq -1 \). Indefinite integrals.

6.6 Estimating definite integrals ......................................................... p. 225
6.7 Integrals involving transcendental functions ........................................ p. 243
Integrals of $y = x^{-1}$. Integrals of exponential functions. Integrals of the hyperbolic sine and cosine functions. Integrals involving trigonometric functions. Integrals of $y = 1/\sqrt{a^2 - x^2}$, and $y = 1/(a^2 + x^2)$.

6.8 Integration by substitution ................................................................. p. 254

6.R Review exercises ................................................................. p. 267

Chapter 7. Additional Applications of Integrals

7.1 More on areas
Areas of regions between graphs $y = f(x)$ and $y = g(x)$ and between curves $x = f(y)$ and $x = g(y)$.

7.2 Volumes by disks and washers
Volumes of solids of revolution by the methods of disks and washers. Using differentials.

7.3 Volumes by slicing
Volumes by the method of slicing. Using differentials.

7.4 Volumes by cylindrical shells
Volumes of solids of revolution by the method of cylindrical shells with $x$- and $y$-integration. Using differentials.

7.5 Lengths of graphs and areas of surfaces of revolution
Lengths of graphs and areas of surfaces of revolution. Using differentials.

7.6 Integrating acceleration and Newton’s law

7.7 Average values of functions
Average values of functions. The Mean Value Theorem for integrals.

7.8 One-dimensional density
One-dimensional density. Weights and centers of gravity of rods.

7.9 Work, energy, and forces of fluids

7.R Review exercises

Chapter 8. Additional Techniques of Integration

8.1. Integration by parts
Integration by parts. Integrals of $y = x^n e^{ax}$, $y = x^n \cos(ax)$, and $y = x^n \sin(ax)$. Integrals involving logarithms, inverse sines, and inverse tangents. Integrals of $y = e^{ax} \sin(bx)$ and $y = e^{ax} \cos(bx)$

8.2. Integration with trigonometric identities
Integrals of $y = \sin^n x \cos^m x$ with $m$ or $n$ odd and $\geq 0$. Integrals of $y = \sin^n x \cos^m x$ with $m$ or $n$ even and $\geq 0$. Integrals of products of $y = \sin(ax)$ and $y = \cos(bx)$. Reduction formulas.
8.3. **Inverse trigonometric substitutions**

8.4 **Integration by partial fractions**

8.5 **Using integral tables and computer/calculator algebra systems**
The direct use of integral tables. Using tables with integration by substitution. Using computer or calculator algebra systems.

8.6 **Improper integrals**
Improper integrals with infinite limits of integration. Improper integrals at vertical asymptotes. Improper integrals involving more than one limit. Normal probability distributions.

8.R **Review exercises**

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**Chapter 9. Differential Equations**

9.1 **Separable first-order equations**
Slope (direction) fields. The differential equation \(dy/dx = ry\) with constant \(r\). The differential equation \(dy/dx = r(x)y\). General separable differential equations. Applications to growth and decay. The Logistic Equation of population growth.

9.2 **More applications of separable equations**

9.3 **Linear first-order equations**
Linear first-order differential equations. Mixing problems.

9.4 **Approximate solutions of first-order equations**

9.5 **The differential equation** \(ay'' + by' + cy = 0\)

9.6 **The differential equation** \(ay'' + by' + cy = f\)
The Method of Undetermined Coefficients. Variation of parameters. Motion of forced springs.

9.R **Review exercises**

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†These programs for a variety of calculators will be available on the web page for the text.
Chapter 10. Sequences and Series

10.1 Infinite sequences
Convergence and divergence of infinite sequences. Bounded and monotone sequences. Least upper and greatest lower bounds.

10.2 Geometric series
Finite geometric series. Infinite geometric series. Infinite repeating decimals.

10.3 The Integral Test

10.4 Comparison Tests
A divergence test. The Comparison Test. The Limit Comparison Test.

10.5 The Ratio Test and alternating series

10.6 Taylor polynomials and Taylor’s Theorem

10.7 Power series

10.R Review exercises

Chapter 11. Conic Sections and Polar Coordinates

11.1 Parabolas, ellipses, and hyperbolas

11.2 Rotation of axes and discriminants
Rotating coordinate axes. Classifying conic sections by their discriminants. Degenerate conic sections. Sections of cones and eccentricity.

11.3 Polar coordinates

11.R Review exercises

Chapter 12. Vectors

12.1 Vectors in the plane
Vectors in a coordinate plane. Adding vectors and multiplying vectors by numbers. Displacement and position vectors. The unit vectors $\mathbf{i}$ and $\mathbf{j}$. Force vectors.

12.2 The dot product in the plane
The dot product and angles between vectors. Perpendicular vectors. The component of one vector in the direction of another. The orthogonal (perpendicular) projection of a vector on a line.

‡Procedures and programs for plotting sequences on a variety of calculators will be available on the web page for the text.
12.3 Vectors in space
Rectangular coordinates in space. The Pythagorean Theorem in space. Vectors in space. The dot product in space. Components and orthogonal projections of vectors in space. The unit vectors \( \mathbf{i}, \mathbf{j}, \) and \( \mathbf{k}. \) Direction cosines and direction angles.

12.4 The cross product
The cross product of two vectors. The scalar triple product. Areas of parallelograms and volumes of parallelepipeds.

12.5 Equations of lines and planes
Parametric equations of lines in space. Perpendicular lines. Equations of planes. The distance from a point to a plane. Skew lines. Intercept equations of planes. Distance from a point to a line in an \( xy \)-plane.

12.R Review exercises

Chapter 13. Vector-Valued Functions

13.1 Parametric equations of curves

13.2 Velocity Vectors

13.3 Acceleration vectors and Newton’s law of motion

13.4 Curvature and acceleration in the plane
Unit tangent and normal vectors to plane curves. Curvature of plane curves. Tangential and normal components of acceleration in the plane. Finding the tangential and normal components of an acceleration vector.

13.5 Curves in space and planetary motion

13.R Review exercises

Chapter 14. Derivatives with Two or More Variables

14.1 Functions of two variables ................................................................. p. 270
The domain, range, and graph of \( z = f(x, y). \) Fixing \( x \) or \( y \): vertical cross sections of graphs. Drawing graphs of functions.

14.2 Horizontal cross sections of graphs and level curves ......................... p. 288
Horizontal cross sections of graphs. Level curves. Estimating function values from level curves. Topographical maps and other contour curves.

14.3 Partial derivatives with two variables ............................................. p. 301
14.4 Chain Rules with two variables .............................................. p. 317
Using the Chain Rule with one variable. The general Chain Rule with two variables. Higher-order partial derivatives.

14.5 Directional derivatives and gradient vectors .............................. p. 327

14.6 Tangent planes and differentials .............................................. p. 342

14.7 Functions of three variables ................................................. p. 354

14.8 Functions of more than three variables ................................... p. 366

14.R Review exercises ................................................................. p. 372

Chapter 15. Maxima and Minima with Two or More Variables

15.1 The First-Derivative Test with two or three variables
Local maxima and minima with two or three variables. The First-Derivative Test with two and three variables. Narrative problems.

15.2 The Second-Derivative Test with two variables
The Second-Derivative Test and Taylor polynomials with one variable. Second-degree Taylor polynomials with two variables. The Second-Derivative Test with two variables.

15.3 Lagrange multipliers
Maxima and minima on curves in the plane and on surfaces in $\mathbb{R}^n$ with $n \geq 3$.

15.R Review exercises

Chapter 16. Multiple Integrals

16.1 Double integrals
Double integrals. Double integrals as volumes. Iterated integrals. Reversing the order of integration. $\Delta x \Delta y$-notation in Riemann sums.

16.2 Applications of double integrals
Volumes of regions between graphs $z = f(x, y)$ and $z = g(x, y)$. Two-dimensional density. Weights and centers of gravity of plates. Average value of $z = f(x, y)$.

16.3 Double integrals in polar coordinates
Double integrals in polar coordinates. Polar curves. Integration techniques (review).

16.4 Triple integrals
Triple integrals in rectangular coordinates. Three-dimensional density and weight. Centers of gravity. Average value of $w = f(x, y, z)$. 
16.5 Triple integrals in cylindrical and spherical coordinates

16.6 Other changes of variables: Jacobians

16.R Review exercises

Chapter 17. Vector Analysis

17.1 Vector fields

17.2 Line integrals
Line integrals in the plane and in space. Interpreting line integrals with respect to arclength. Relating $dx, dy,$ and $dz$ to $ds$. Negatives, sums and differences of curves. Average value of a function on a curve.

17.3 Work and flux
Work done by a constant force field along a line segment. Work done by a variable force field along a curve. Unit normal vectors to oriented plane curves. Flux of a constant velocity field across a line segment. Flux of a variable velocity field across an oriented curve.

17.4 Path independent line integrals
Path independent line integrals and conservative force fields in two and three dimensions.

17.5 The Divergence and Stokes' Theorems in the plane

17.6 Surface integrals
Surface integrals. Average values on surfaces. Weights and centers of gravity of surfaces.

17.7 The Divergence and Stokes' Theorems in space

17.R Review exercises