Hop-spanners for geometric intersection graphs

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Abstract

A t-spanner of a graph $G = (V, E)$ is a subgraph $H = (V, E')$ that contains a uv-path of length at most t for every $uv \in E$. It is known that every n-vertex graph admits a $(2k-1)$-spanner with $O(n^{1+1/k})$ edges for $k \geq 1$. This bound is the best possible for $1 \leq k \leq 9$ and is conjectured to be optimal due to Erdős' girth conjecture.

We study t-spanners for $t \in \{2, 3\}$ for geometric intersection graphs in the plane. These spanners are also known as *t-hop spanners* to emphasize the use of graph-theoretic distances (as opposed to Euclidean distances between the geometric objects or their centers). We obtain the following results: (1) Every n-vertex unit disk graph (UDG) admits a 2-hop spanner with $O(n)$ edges; improving upon the previous bound of $O(n \log n)$. (2) The intersection graph of n axis-aligned fat rectangles admits a 2-hop spanner with $O(n \log n)$ edges, and this bound is tight up to a factor of $\log \log n$. (3) The intersection graph of n fat convex bodies in the plane admits a 3-hop spanner with $O(n \log n)$ edges. (4) The intersection graph of n axis-aligned rectangles admits a 3-hop spanner with $O(n \log^2 n)$ edges.

(Joint worth with Jonathan B. Conroy)