Edge-ordered graphs with linear and almost linear extremal functions

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Abstract

As an extension of Turán-type classical extremal graph theory we consider simple graphs with a linear order on its edges (edge-ordered graphs) and ask how many edges can such a graph have (as a function of the number of vertices, the extremal function of the forbidden pattern) if we forbid a fixed subgraph with a fixed edge-order.

In the classical theory forests have linear extremal functions, while every non-forest has extremal function $\Omega(n^c)$ for some $c > 1$. This dichotomy fails for edge-ordered graphs, so one can ask two separate questions: what edge-ordered graphs have linear extremal functions and what edge-ordered graphs almost linear extremal functions, i.e., $n^{1+o(1)}$. We give a complete characterization for the latter problem and also solve the former problem for connected edge-ordered graphs. The analogous question for vertex-ordered graphs (characterizing the ones with almost linear extremal functions) is subject to along standing conjecture and is still open.

This is joint work with Gaurav Kucheriya.