From selection theorems to weak epsilon-nets (and back?)

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Abstract

Given a finite set P in the Euclidean d-space, we say that N is a weak ϵ-net if it pierces all the convex sets that encompass at least $\epsilon|P|$ points of P.

We lay out the recent construction of $o(\epsilon^{-d+1/2})$-size weak epsilon-nets in dimension 3 and higher, thereby improving upon the 30 year old result of Chazelle, Clarkson, Edelsbrunner, Grigni, Guibas, ans Sharir.

The construction is based on a remarkable reduction to the so called second selection problem which concerns piercing many simplices in a dense geometric hypergraph with a single point. If time permits, we discuss the prospect of improving the second selection theorem of Alon, Barany, Furedi, and Kleitman, which would bring about a better bound for the notorious k-set problem in dimension 5 and higher.