Practice Final

Problem 1. Construct the general solution of $x' = \begin{pmatrix} -7 & 10 \\ -4 & 5 \end{pmatrix} x$ involving complex eigenfunctions and then obtain the general real solution.

Problem 2. Find the distance between the vector $y = \begin{pmatrix} 5 \\ -9 \\ 5 \end{pmatrix}$ and the subspace

$$W = \left\{ \begin{pmatrix} -3 \\ -5 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} \right\}.$$

Then find an orthonormal basis for W.

Problem 3. Find an orthogonal basis for the column space of $A = \begin{pmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{pmatrix}$.

Problem 4. Find the orthogonal projection of $b = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ onto the column space of $A = \begin{pmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{pmatrix}$.

Problem 5. Let W be a subspace of $\mathbb{R}^n$. Prove that $W^\perp$ is a subspace.

Problem 6. Let $\mathcal{B} = \{\begin{pmatrix} -1 \\ 8 \end{pmatrix}, \begin{pmatrix} 1 \\ -5 \end{pmatrix}\}$ and $\rfloor = \{\begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}\}$. Find the change of coordinate matrix from $\mathcal{C}$ to $\mathcal{B}$.

Problem 7. Let $A = \begin{pmatrix} 5 & -2 \\ 1 & 3 \end{pmatrix}$. Find an invertible matrix P and a matrix C of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, such that $A = PCP^{-1}$.

Problem 8. Diagonalize $A = \begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix}$, that is, find D and P such that $A = PDP^{-1}$. Then compute A^8.

Problem 9. Find the inverse of the matrix $A = \begin{pmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{pmatrix}$.

Problem 10. Let $T : R^2 \to R^2$ be linear transformation that first reflects points through the x_1-axis, and then reflects points points through the x_2-axis. Find the standard matrix of T.

Problem 11. Given $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$, find $\begin{vmatrix} 3a & 3b & 3c \\ 2d+g & 2e+h & 2f+i \\ g & h & i \end{vmatrix}$.

Problem 12. Let $H \subset \mathbb{P}_4$ be the set of all polynomials of degree at most 4 such that $p(0) = 3$. Is H a subspace? Why or why not?