Practice Test 1

Problem 1. Find an LU factorization of the matrix $A = \begin{bmatrix} -5 & 3 & 4 \\ 10 & -8 & -9 \\ 15 & 1 & 2 \end{bmatrix}$. Solve for $\mathbf{x}$ in the matrix equation $A\mathbf{x} = \mathbf{0}$.

Problem 2. Is the matrix $A = \begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix}$ invertible? If so, find A^{-1}.

Problem 3. Solve the matrix equation $A\mathbf{x} = \mathbf{0}$ where $A = \begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$. Are the columns of A independent? Explain your answer.

Problem 3. Let A be an $n \times n$ matrix. If the columns of A are linearly independent, then are the rows of A linearly independent? Explain your answer.

Problem 4. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$, then what is $\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix}$

Problem 5. Is the vector $b = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ a linear combination of the columns $A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}$? Explain why.

Problem 6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, such that T first reflects points through the vertical x_2-axis and then rotates (counterclockwise) points $\pi/2$ radians. Find the standard matrix A that represents T.