Math 20A
October 23, 2017

Midterm Exam 1 v. B
(Total Points: *25*)

Name: Key 2
PID:

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Instructions

1. Write your Name and PID in the spaces provided above.
2. Make sure your Name is on every page.
3. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
4. Put away ANY devices that can be used for communication or can access the Internet.
5. You may use one handwritten page of notes, but no books or other assistance during this exam.
6. Read each question carefully and answer each question completely.
7. Write your solutions clearly in the spaces provided.
8. Show all of your work. No credit will be given for unsupported answers, even if correct.

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(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. The line tangent to the curve $y = 9 - x^2$ at the point $(3, 0)$ also passes through the point $(0, b)$. Find $b$.

$y' = -2x$

tangent line

$y - y_0 = m(x - x_0)$

$x_0 = 3$

$y_0 = 0$

$m = y'\big|_{x=3} = -6$

$\boxed{y = -6(x-3)}$

$\boxed{b = -6(-3) = 18}$

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(6 points) 2. Evaluate the following limits:

(a) $\lim_{x\to\infty} \frac{x^2 - 5x + 6}{x^2 - 2x - 3}$ $= \lim_{x\to\infty} \frac{x^2}{x^2} = \boxed{1}$

higher order terms

(b) $\lim_{x\to 3} \frac{x^2 - 5x + 6}{x^2 - 2x - 3}$ $= \lim_{x\to 3} \frac{\cancel{(x-3)}(x-2)}{\cancel{(x-3)}(x+1)}$

$= \boxed{\frac{1}{4}}$

(c) $\lim_{x\to 0} \frac{\sin(4x)}{\sin(3x)}$ (Hint: You may use the fact that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$.)

$= \lim_{x\to 0} \underbrace{\left(\frac{\sin 4x}{4x}\right)}_{1} 4\cancel{x} \underbrace{\left(\frac{3x}{\sin(3x)}\right)}_{1} \frac{1}{3\cancel{x}}$

by hint

$= \boxed{\frac{4}{3}}$

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(6 points) 3. Show that the equation $x^4 + x - 3 = 0$ has at least two solutions in the interval $(-2, 2)$.

$f(x) = x^4 + x - 3$

$f(-2) = 16 - 2 - 3 = 11$ $+$

f is a polynomial $\Rightarrow$ f is continuous in $[-2, 2]$

$f(0) = -3$ $-$

$f(2) = 16 + 2 - 3 = 15$ $+$

by IVT, f has a root in $(-2, 0)$ & $(0, 2)$.

Hence $x^4 + x - 3 = 0$ has at least 2 solutions in $(-2, 2)$

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(6 points) 4. Let $f(u) = u^3$. Use the definition of the derivative to find $f'(u)$ for any value of $u$. (Note: Be sure to use the definition of the derivative. Applying the power rule will not earn any credit.)

$$f'(u) = \lim_{h \to 0} \frac{f(u+h) - f(u)}{h}$$

$$= \lim_{h \to 0} \frac{(u+h)^3 - u^3}{h}$$

$$= \lim_{h \to 0} \frac{u^3 + 3u^2h + 3uh^2 + h^3 - u^3}{h}$$

$$= \lim_{h \to 0} 3u^2 + 3uh + h^2$$

$$= \boxed{3u^2}$$