Department of Mathematics,
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Computational Geometric Mechanics Research Seminar

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Geometric Methods for Adjoint Systems

Abstract:

Adjoint systems are widely used to inform control, optimization, and design in systems described by ordinary differential equations or differential-algebraic equations. In this session, we explore the geometric properties and develop methods for such adjoint systems. In particular, we utilize symplectic and presymplectic geometry to investigate the properties of adjoint systems associated with ordinary differential equations and differential-algebraic equations, respectively. We show that the adjoint variational quadratic conservation laws, which are key to adjoint sensitivity analysis, arise from (pre)symplecticity of such adjoint systems. We discuss various additional geometric properties of adjoint systems, such as symmetries and variational characterizations. For adjoint systems associated with a differential-algebraic equation, we relate the index of the differential-algebraic equation to the presymplectic constraint algorithm of Gotay and Nester. As an application of this geometric framework, we discuss how the adjoint variational quadratic conservation laws can be used to compute sensitivities of terminal or running cost functions. Furthermore, we develop structure-preserving numerical methods for such systems using Galerkin Hamiltonian variational integrators which admit discrete analogues of these quadratic conservation laws. We additionally show that such methods are natural, in the sense that reduction, forming the adjoint system, and discretization all commute, for suitable choices of these processes. We utilize this naturality to derive a variational error analysis result for the presymplectic variational integrator that we use to discretize the adjoint DAE system. This is joint work with Prof. Melvin Leok.

In the post-talk discussion session, we plan to discuss future directions; in particular, exploring the geometry of adjoint systems for infinite-dimensional spaces with the application of PDE-constrained optimization in mind.

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