Math 10A Midterm 2
February 27, 2009
Notes allowed (one page), graphing calculators allowed, show all work
50 points total

1. (20 points, 5 points each) Find the derivatives of the following functions. *Do not simplify derivatives.*
   a. \( g(t) = \sin^2 t + \cos(t^2) \). Chain rules. \( g'(t) = \cos t \cdot 2 \sin t + 2t \cdot (-\sin(t^2)) \).
   b. \( f(x) = e^{x^2} \cos(2x) \). Product rule, chain rules.
      \[
      2x \cdot e^{x^2} \cdot \cos(2x) + e^{x^2} \cdot 2 \cdot (-\sin(2x)).
      \]
   c. \( w(x) = \sqrt{\frac{\ln x - x}{e^{2x}}} \). Chain rule, quotient rule, chain rule.
      \[
      w'(x) = \left( \frac{1}{x-1} \cdot e^{2x} - (\ln x - x) \cdot \frac{2e^{2x}}{e^{2x}} \right) \cdot \frac{1}{2 \sqrt{\frac{\ln x - x}{e^{2x}}}}.
      \]
   d. Compute \( \frac{dy}{dx} \) for \( x^3 + xy + e^y = y^2 + 1 \). Implicit differentiation.
      \[
      3x^2 + y + xy' + y' \cdot e^y = 2yy'.
      \]
      Solve for \( y' \). \( y' = \frac{3x^2 + y}{2y - x - e^y} \).

2. (10 points) Given the function \( y = \sin(2x) - \cos(x) \).
   a. (8 points) Find the tangent line of \( y \) where \( x = 0 \). Find \( y' \).
      \[
      y' = 2 \cos(2x) + \sin x.
      \]
      Evaluate \( y'(0) \). \( y'(0) = 2 \cos(0) + \sin 0 = 2 \). Find tangent line.
      \[
      y = 2(x-0) - 1 = 2x - 1.
      \]
   b. (2 points) Use the tangent line to approximate the value of \( y \) at \( x = 0.1 \).
      Evaluate \( y(0.1) \) using tangent line from part (a).
      \[
      y(0.1) = 2(0.1) - 1 = -0.8.
      \]

3. (10 points) For what intervals is \( f(x) = x^4 - 6x^2 \) both increasing and concave down. \( f'(x) = 4x^3 - 12x = 4x(x^2 - 3) = 4x(x - \sqrt{3})(x + \sqrt{3}) \).
   \( f'(x) \) is increasing when \( f'(x) > 0 \) or when \(-\sqrt{3} < x < 0 \) and \( \sqrt{3} < x \).
   \[
   f''(x) = 12x^2 - 12 = 12(x^2 - 1) = 12(x - 1)(x + 1).
   \]
   \( f''(x) < 0 \) or when \(-1 < x < 1 \). Therefore, \( f(x) \) is increasing and concave down when \(-1 < x < 0 \).
4. (10 points) Given \( g(x) = xe^{-x} \) for \( x \geq 0 \). Find and classify the critical point(s) of \( g \) as local maxima or local minima or neither. Find critical point.

\[
g'(x) = e^{-x} + x \cdot (-1)(e^{-x}) = e^{-x}(1-x).
\]

Solve \( e^{-x}(1-x) = 0 \). Therefore, the only critical point is \( x = 1 \). Use either the First Derivative Test or the Second Derivative Test. (FDT) Since \( e^{-x} > 0 \), \( g'(x) > 0 \) to the left of \( x = 1 \), and \( g'(x) < 0 \) to the right of \( x = 1 \). Therefore, \( g'(x) \) changes from positive to negative at \( x = 1 \), and \( x = 1 \) is a local maximum of \( g(x) \).