Math 10A Midterm 2.
February 27, 2009
Notes allowed (one page), graphing calculators allowed, show all work
50 points total

1. (20 points, 5 points each) Find the derivatives of the following functions. Do not simplify derivatives.
   a. \( g(t) = \cos^2 t + \sin\left(t^2\right) \). Chain rules. \( g'(t) = -\sin t \cdot 2 \cos t + 2t \cdot \cos\left(t^2\right) \).
   b. \( f(x) = e^{x^2} \sin(2x) \). Product rule, chain rules.
      \( 2x \cdot e^{x^2} \cdot \sin(2x) + e^{x^2} \cdot 2 \cos(2x) \).
   c. \( w(x) = \frac{x - \ln x}{e^{2x}} \). Chain rule, quotient rule, chain rule.
      \( w'(x) = \frac{1 - \frac{1}{x} \cdot e^{2x} - (x - \ln x) \cdot 2 \cdot e^{2x}}{(e^{2x})^2} \cdot \frac{1}{2 \sqrt{\frac{x - \ln x}{e^{2x}}} \cdot \frac{1}{e^{2x}}} \).
   d. Compute \( dy/dx \) for \( x^2 + xy + e^y = y^3 + 1 \). Implicit differentiation.
      \( 2x + y + xy' + y' \cdot e^y = 3y^2y' \). Solve for \( y' \).
      \( y' = \frac{2x + y}{3y^2 - x - e^y} \).

2. (10 points) Given the function \( y = \sin(3x) + \cos(x) \).
   a. (8 points) Find the tangent line of \( y \) where \( x = 0 \). Find \( y' \).
      \( y' = 3 \cos(3x) - \sin x \). Evaluate \( y'(0) \).
      \( y'(0) = 3 \cos(0) - \sin 0 = 3 \).
      Find tangent line.
      \( y = 3(x - 0) + 1 = 3x + 1 \).
   b. (2 points) Use the tangent line to approximate the value of \( y \) at \( x = 0.1 \).
      Evaluate \( y(0.1) \) using tangent line from part (a).
      \( y(0.1) = 3(0.1) + 1 = 1.3 \).

3. (10 points) For what intervals is \( f(x) = x^4 - 6x^2 \) both decreasing and concave down. \( f'(x) = 4x^3 - 12x = 4x(x^2 - 3) = 4x\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) \).
   \( f(x) \) is decreasing when \( f'(x) < 0 \) or when \( x < -\sqrt{3} \) and \( 0 < x < \sqrt{3} \).
   \( f''(x) = 12x^2 - 12 = 12(x^2 - 1) = 12(x - 1)(x + 1) \). \( f(x) \) is concave down when \( f''(x) < 0 \) or when \(-1 < x < 1 \). Therefore, \( f(x) \) is decreasing and concave down when \( 0 < x < 1 \).
4. (10 points) Given \( f(x) = \frac{x}{e^x} \) for \( x \geq 0 \). Find and classify the critical point(s) of \( f \) as local maxima or local minima or neither. Find critical point.

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g'(x) = \frac{1 \cdot e^x - x \cdot e^x}{(e^x)^2} = \frac{1 - x}{e^x}. \]

Solve \( \frac{1 - x}{e^x} = 0 \). The only critical point is \( x = 1 \).

Use either the First Derivative Test or the Second Derivative Test. (FDT) Since \( e^x > 0 \), \( g'(x) > 0 \) to the left of \( x = 1 \), and \( g'(x) < 0 \) to the right of \( x = 1 \). Therefore, \( g'(x) \) changes from positive to negative at \( x = 1 \), and \( x = 1 \) is a local maximum of \( g(x) \).