**MATH 180A - INTRODUCTION TO PROBABILITY**
**PRACTICE MIDTERM #2**

FALL 2018

Name (Last, First): 
Student ID: 
TA: 

<table>
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<tr>
<th>SO AS TO NOT DISTURB OTHER STUDENTS, EVERY-ONE MUST STAY UNTIL THE EXAM IS COMPLETE.</th>
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<tbody>
<tr>
<td>ANSWERS TO THE TRUE/FALSE QUESTIONS DO NOT NEED TO BE JUSTIFIED; HOWEVER, INCORRECT ANSWERS TO THE TRUE/FALSE QUESTIONS ARE PENALIZED. IN PARTICULAR, A CORRECT ANSWER IS WORTH 5 POINTS, AN INCORRECT ANSWER IS WORTH -5 POINTS, AND A BLANK ANSWER IS WORTH 0 POINTS.</td>
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<td>REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT.</td>
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<td>THIS EXAM WILL BE SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED.</td>
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<td>THE ACTUAL MIDTERM EXAM CONSISTS OF 5 TRUE/FALSE QUESTIONS AND 3 LONGER FORMAT QUESTIONS. YOUR ANSWERS TO THE LONGER FORMAT QUESTIONS SHOULD BE CAREFULLY JUSTIFIED. YOU ARE ALLOWED TO USE RESULTS FROM THE TEXTBOOK, HOMEWORK, AND LECTURE.</td>
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1. (25 points) Label the following statements as true or false. Any ambiguous answer (for example, resembling a hybrid of T and F) will be treated as an incorrect answer.

(a) ________ The median of a continuous random variable is unique.

(b) ________ If $X \sim \text{Exp}(\lambda)$ and $s, t > 0$, then $\mathbb{P}(X < s \mid X < s + t) = \mathbb{P}(X < t)$.

(c) ________ If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $\mathbb{E}[X^2] = \sigma^2 + \mu^2$.

(d) ________ If a random variable is not discrete, then it is continuous.

(e) ________ If $X$ is a continuous random variable and $F_X$ is its CDF, then for $s, t \in \mathbb{R}$ such that $s < t$, we have the equality $F_X(t) - F_X(s) = \mathbb{P}(s < X < t)$.

(f) ________ If $X \sim \text{Exp}(\lambda)$ and $Y \sim \text{Exp}(\eta)$ are independent, then $\min(X, Y) \sim \text{Exp}(\lambda + \eta)$. 
2. (25 points) You are trying to get to a party, which is 30 minutes away by bus. Suppose that the waiting time for a bus is distributed according to an exponential distribution with a rate of two buses every hour. If you wait longer than 30 minutes for a bus, you decide to give up and take a taxi to the party. The taxi ride to the party only takes 15 minutes. Let $Y$ be the amount of time it takes for you to get to the party, which includes BOTH the wait time and the travel time.

   (a) (15 pts) Calculate the CDF of $Y$. You may choose to write $Y$ in terms of hours or in terms of minutes.
(b) (10 pts) Calculate the expected value $\mathbb{E}[Y]$. Again, you may choose to write $Y$ in terms of hours or in terms of minutes.
3. Let $X$ and $Y$ be independent continuous random variables with CDFs $F_X$ and $F_Y$ respectively. We also use the notation $f_X$ and $f_Y$ to denote their respective densities.

(a) (10 pts) Compute the density of the random variable $Z = \min(X, Y) = \begin{cases} X & \text{if } X \leq Y \\ Y & \text{if } X > Y \end{cases}$ in terms of the functions $F_X, F_Y, f_X, f_Y$. 
(b) (15 pts) Compute the density of the random variable $Z = X + Y$. Your answer should be in terms of the densities $f_X$ and $f_Y$ of $X$ and $Y$ respectively.
4. Let $P = (X,Y)$ be a uniform random point in the rectangle

$$R = [0, 1] \times [0, 2] = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 2\}.$$

(a) (10 pts) What is the joint density $f_{(X,Y)}(x, y)$ of $P$? What is the marginal density $f_X(x)$ of $X$? What is the marginal density $f_Y(y)$ of $Y$? Are $X$ and $Y$ independent?
(b) (15 pts) Let $Z = X + Y$ be the sum of the coordinates of the random point $P$. Determine the CDF of $Z$. Compute the expectation $E[Z]$. 
5. Suppose that the joint density of the random variables $X, Y$ is given by the function

$$f_{(X,Y)}(x, y) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-y)^2}{2}} & \text{if } x \in \mathbb{R} \text{ and } y > 0, \\ 0 & \text{otherwise}. \end{cases}$$

Find the marginal density of $X$. Your answer should be in terms of the CDF $\Phi$ of the standard normal distribution.
(ADDITIONAL SPACE FOR WORK, CLEARLY INDICATE THE PROBLEM YOU ARE WORKING ON)
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