1. $f(x,y) = (x^2-y)e^y$

$f_x(x,y) = 2xe^y = 0 \Rightarrow x = 0$ by the hint

$f_y(x,y) = -e^y + (x^2-y)e^y \Rightarrow e^y(-1-y) = 0 \Rightarrow y = -1$

since $x=0$

again, by the hint

$(0,-1)$ is the only critical point

$f_{xx}(x,y) = 2e^y$

$f_{xy}(x,y) = 2xe^y$

$f_{yy}(x,y) = -e^y - e^y + (x^2-y)e^y$

@ $(0,-1)$

$f_{xx}(0,1) = 2e^{-1}$

$f_{xy}(0,1) = 0$

$f_{yy}(0,1) = -e^{-1}$

$D = (2e^{-1})(-e^{-1}) - 0^2 < 0$

so $(0,-1)$ is a saddle point

2. $f(x,y) = x^2 - x - y^2 + 4y - 1$

$f_x(x,y) = 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

$f_y(x,y) = -2y + 4 = 0 \Rightarrow y = 2$

$(\frac{1}{2}, 2)$ is not in the domain D

so, max and min occur on the boundary of D

$y=1,\ x \in [0,1] \longrightarrow f(x,1) = x^2 - x + 2$

$x=0,\ y \in [0,1]$ $\boxed{D}$ $x=1,\ y \in [0,1] \longrightarrow f(1,y) = -y^2 + 4y - 1$

$y=0$
$x \in [0,1]$

$\downarrow$

$f(x,0) = x^2 - x - 1$

$x=0,\ y \in [0,1] \longrightarrow f(0,y) = -y^2 + 4y - 1$, same as $f(1,y) = -y^2 + 4y - 1$

$f(1,y) = -y^2+4y-1$ for $y \in [0,1]$

$f_y(1,y) = -2y+4 = 0 \Rightarrow y=2$, critical point outside domain

$f(1,0) = -1$, $f(1,1) = 2$

$f(x,1) = x^2-x+2$ for $x \in [0,1]$

$f_x(x,1) = 2x-1 = 0 \Rightarrow x = \frac{1}{2}$

$f(\frac{1}{2},1) = \frac{1}{4} - \frac{1}{2} + 2 = \frac{7}{4}$, $f(0,1) = 2$, $f(1,1) = 2$

$f(x,0) = x^2-x-1$

same as previous case but subtract 3

$f(\frac{1}{2},0) = \frac{-5}{4}$, $f(0,0) = -1$, $f(1,0) = -1$

So, abs max is $f(1,1) = f(0,1) = 2$

abs min is $f\left(\frac{1}{2}, 0\right) = \frac{-5}{4}$

3. $f(x,y,z) = x^{-3} + y^{-3} + z^{-3} + \pi^{-3}$

$g(x,y,z) = x^{-4} + y^{-4} + z^{-4} = 1$

$\nabla f = \lambda \nabla g \Rightarrow$

$$-3x^{-4} = \lambda(-4x^{-5})$$
$$-3y^{-4} = \lambda(-4y^{-5})$$
$$-3z^{-4} = \lambda(-4z^{-5})$$

$\Rightarrow$
$$\lambda = \frac{3}{4}x$$
$$\lambda = \frac{3}{4}y \quad \Rightarrow \quad x = y = z \quad \Rightarrow \quad 3 = x^4$$
$$\lambda = \frac{3}{4}z \qquad\qquad\qquad \Rightarrow \quad x = \pm\sqrt[4]{3}$$

max is $f(\sqrt[4]{3}, \sqrt[4]{3}, \sqrt[4]{3}) = \dfrac{3}{3^{3/4}} + \dfrac{1}{\pi^3}$

min is $f(-\sqrt[4]{3}, -\sqrt[4]{3}, -\sqrt[4]{3}) = \dfrac{-3}{3^{3/4}} + \dfrac{1}{\pi^3}$

4. $$\iint_R \frac{1}{\sqrt{2x+y}}\,dA = \int_0^1 \int_0^2 \frac{1}{\sqrt{2x+y}}\,dy\,dx$$

$$\int_0^2 \frac{1}{\sqrt{2x+y}}\,dy = 2\sqrt{2x+y}\,\Big|_0^2 = 2\sqrt{2x+2} - 2\sqrt{2x}$$

$$\int_0^1 2\sqrt{2x+2} - 2\sqrt{2x}\,dx = 2\,\frac{2}{3}\,\frac{(2x+2)^{3/2}}{2} - 2\,\frac{2}{3}\,\frac{(2x)^{3/2}}{2}\,\Big|_0^1$$

$$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(2)^{3/2} - \frac{2}{3}(2)^{3/2}$$

$$= \frac{16}{3} - \frac{4}{3}\,2^{3/2}$$