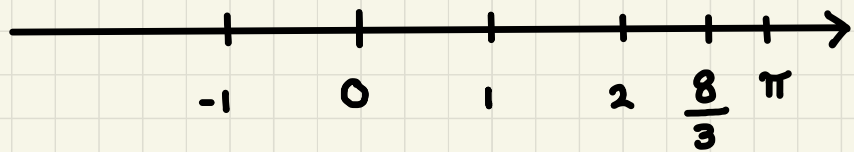
Three-dimensional coordinate systems

Notation

$\mathbb{R}$ is the set of real numbers

What is a real number?

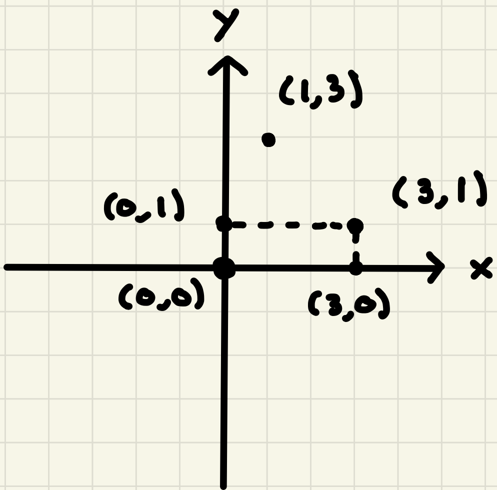
$\mathbb{R}$ "=" 

This visual representation is called the real line for the obvious reason

$\mathbb{R}^2$ is the set of ordered pairs of real numbers

Formally, $\mathbb{R}^2 = \{(a, b) : a, b \in \mathbb{R}\}$

For example, $(3, 1) \neq (1, 3)$

$\mathbb{R}^2$ "=" 

This visual representation of $\mathbb{R}^2$ is called the plane

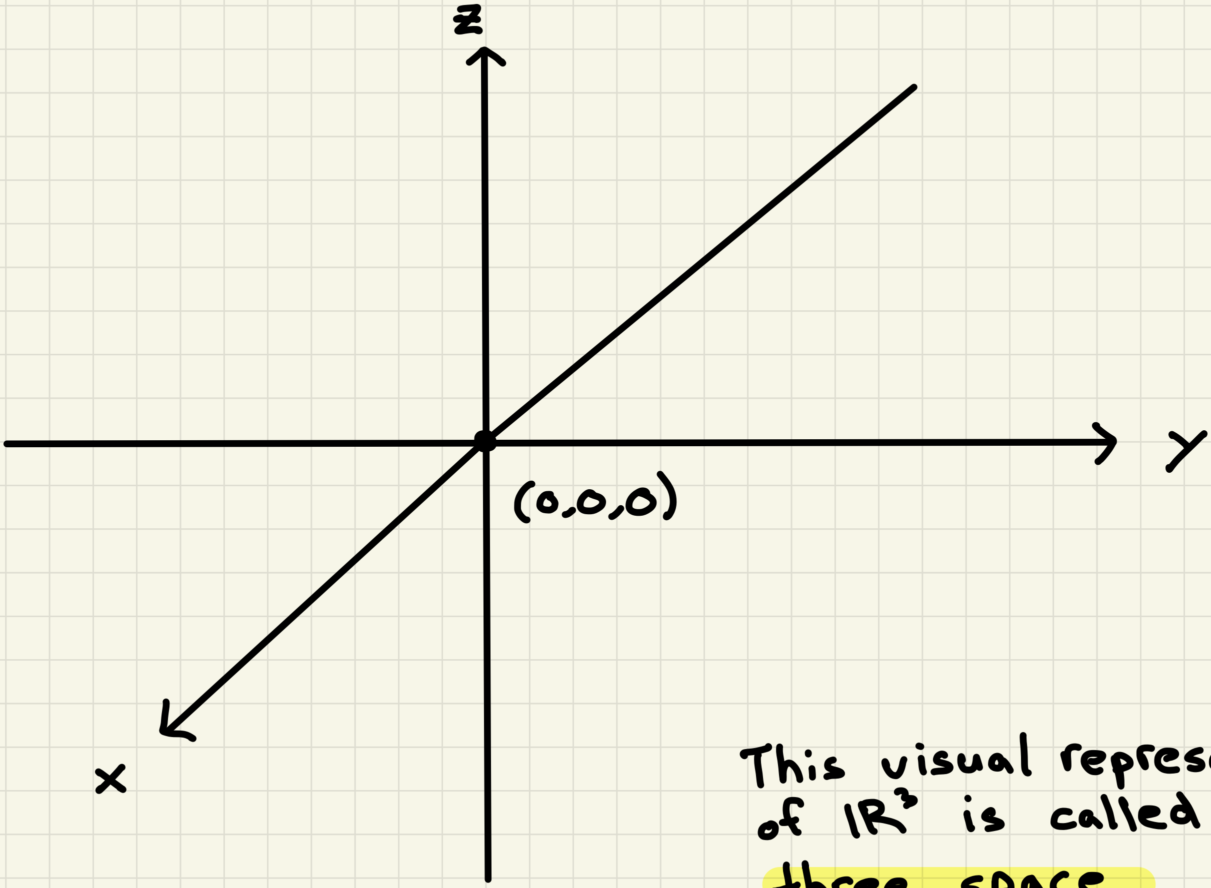
More generally, $\mathbb{R}^n = \{(a_1, \ldots, a_n) : a_1, \ldots, a_n \in \mathbb{R}\}$

$\mathbb{R}^n$ "=" ?

We will consider the case of $n = 3$

Formally, $\mathbb{R}^3 = \{\underbrace{(a, b, c)}_{\text{ordered triple}} : a, b, c \in \mathbb{R}\}$

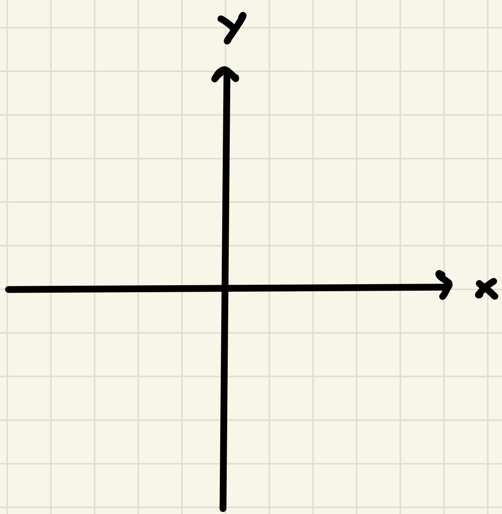
$$\mathbb{R}^3 = \{(a, b, c) : a, b, c \in \mathbb{R}\}$$

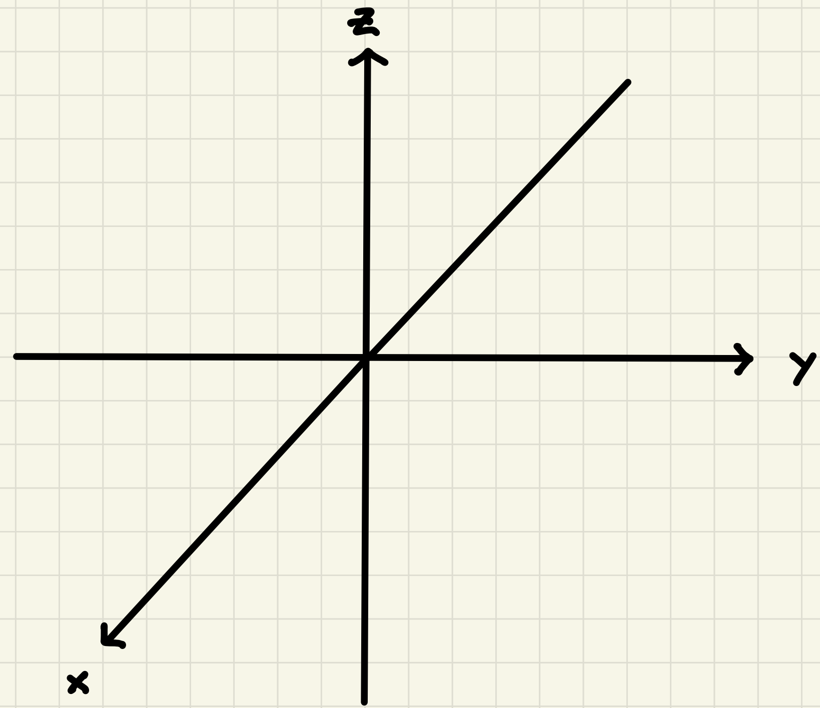


This visual representation of $\mathbb{R}^3$ is called three space

Graphing equations

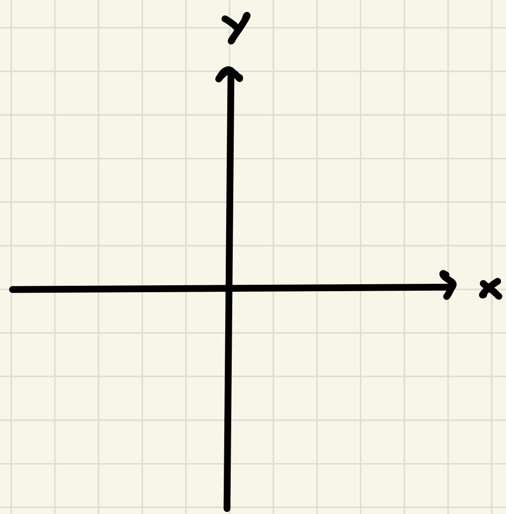
$x = 2$

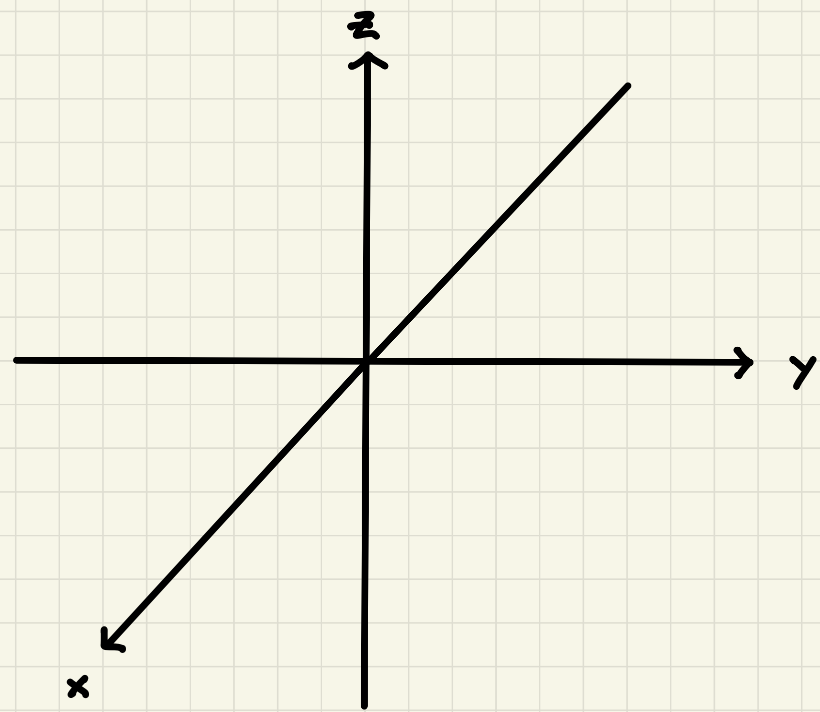




Graphing equations

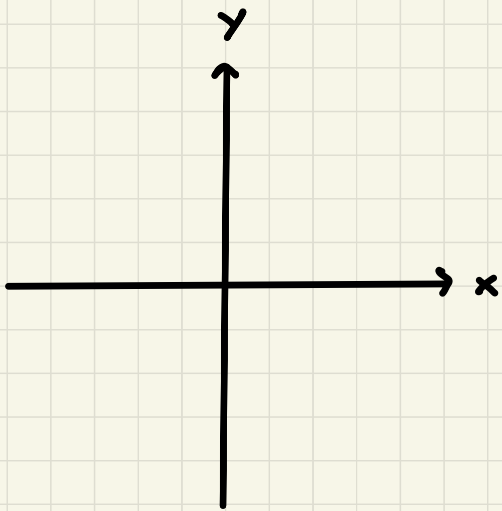
$y = -5$

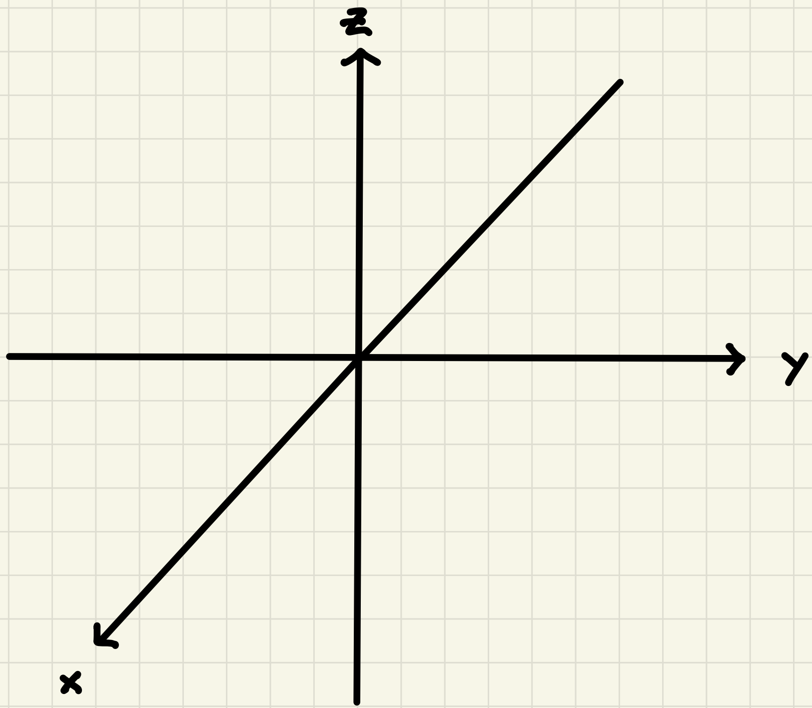




Graphing equations

$z = \pi^{\pi}$

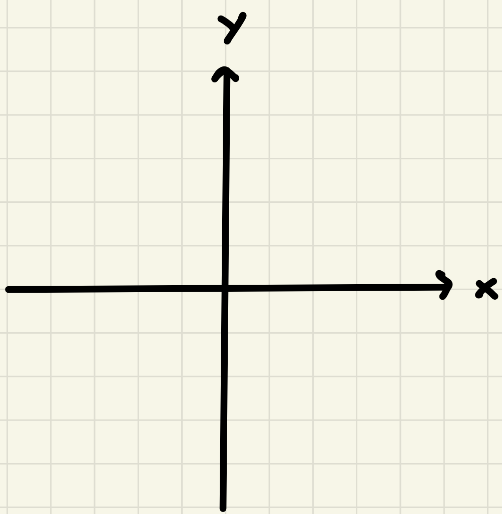




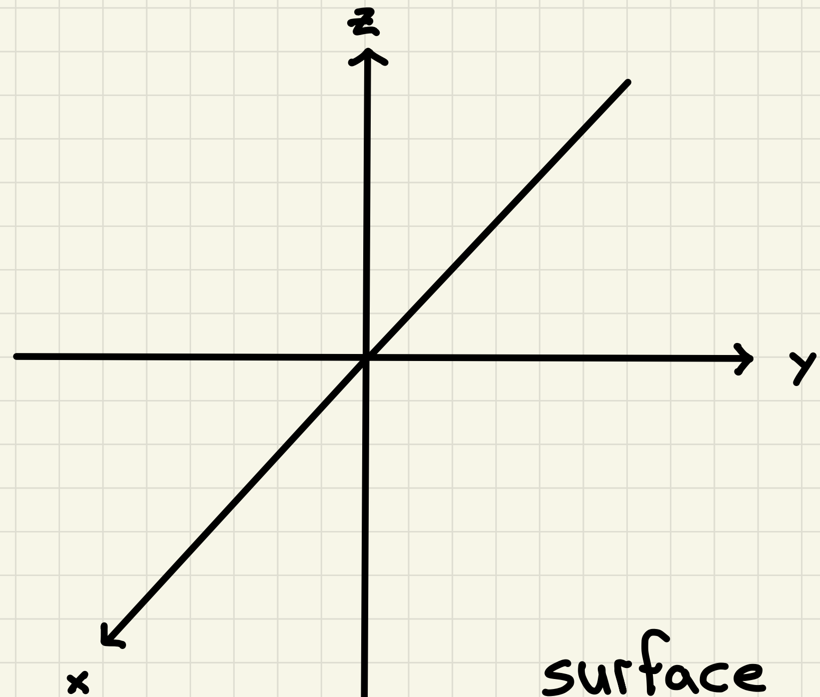
Moral: anything not specified is a free variable

Graphing equations

$y = x^2$



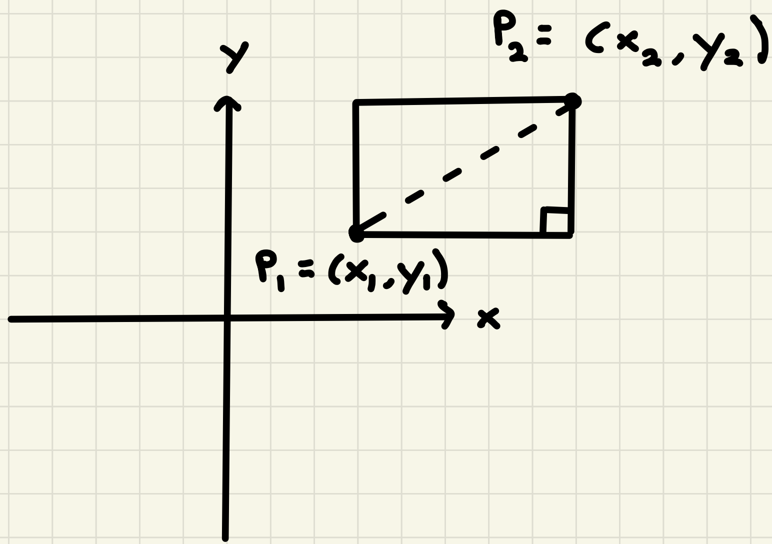
curve



surface

Distance between points

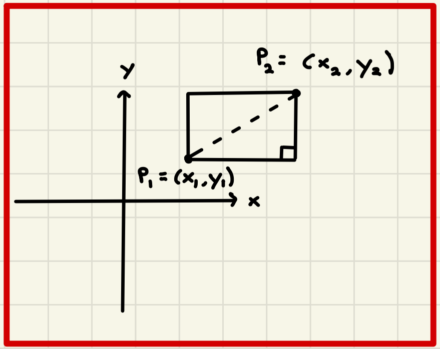
in the plane $\mathbb{R}^2$

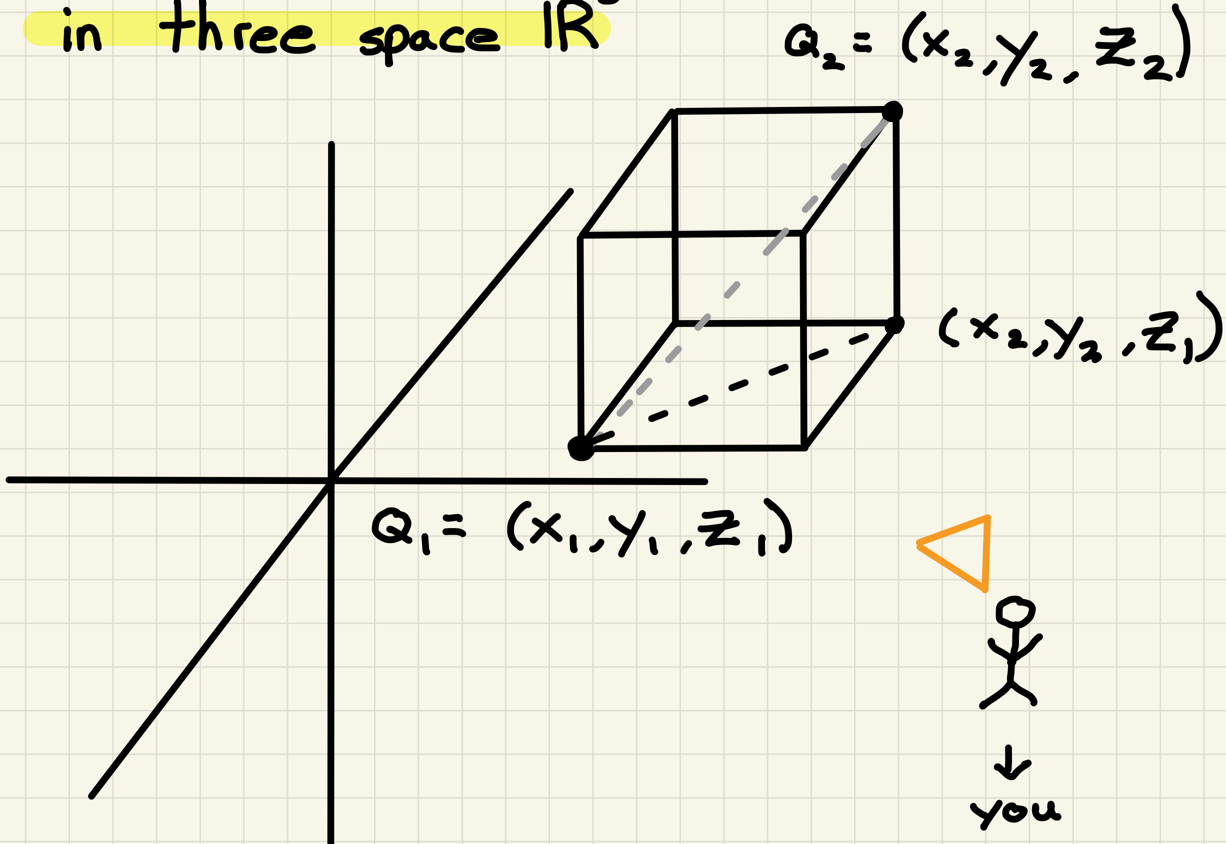


$$|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= |P_2P_1|$$

Distance between points

in three space $\mathbb{R}^3$

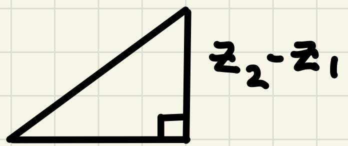




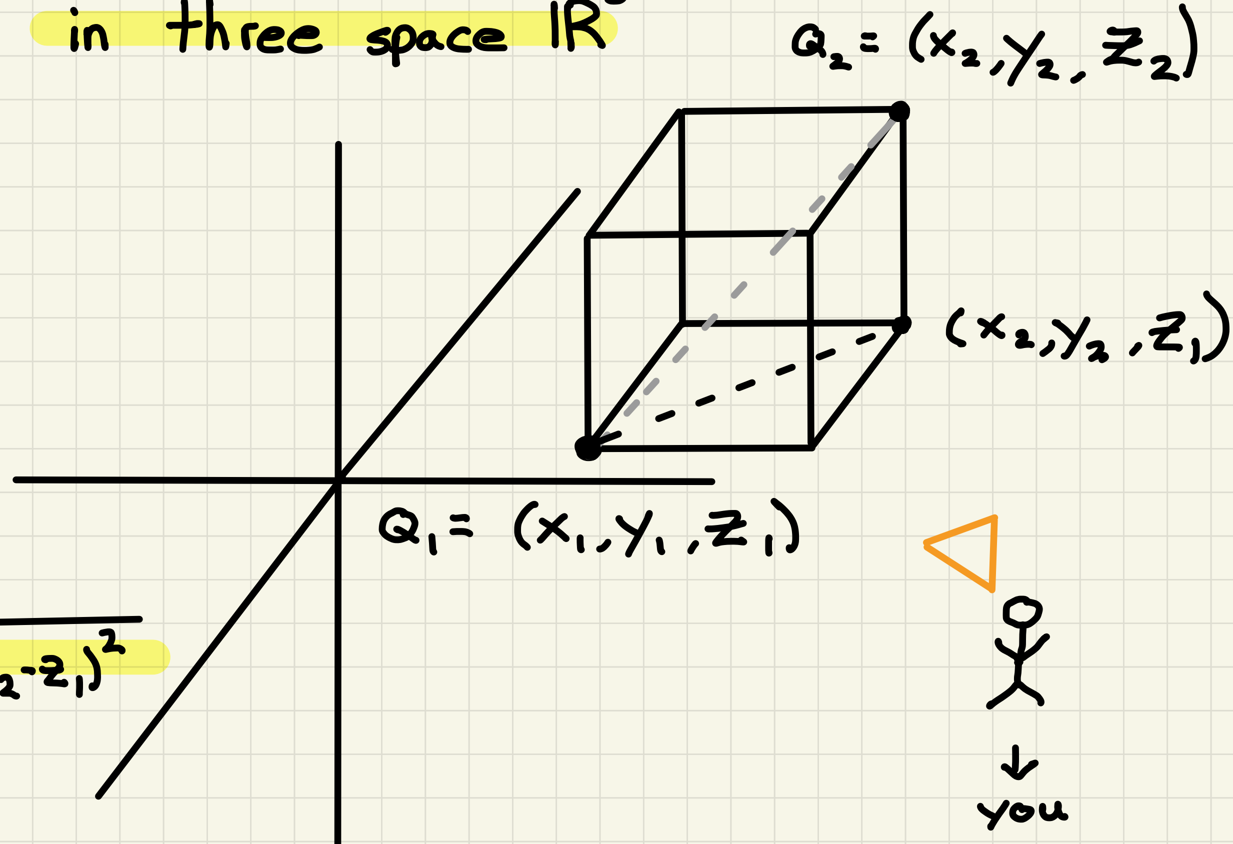
Distance between points

in three space $\mathbb{R}^3$

What you see



$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

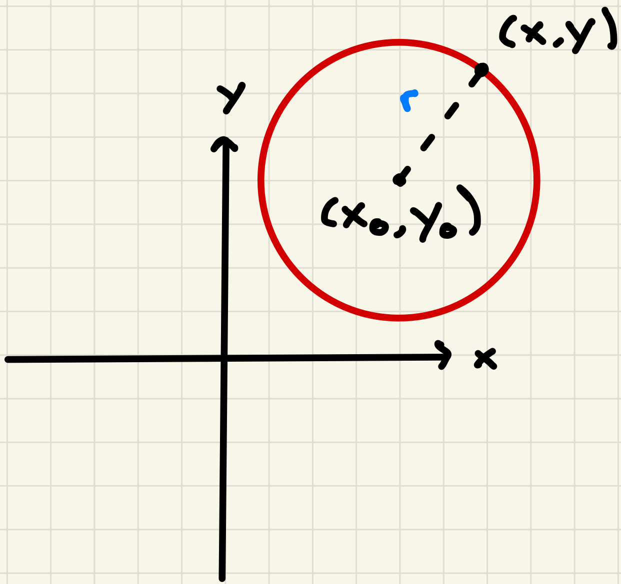


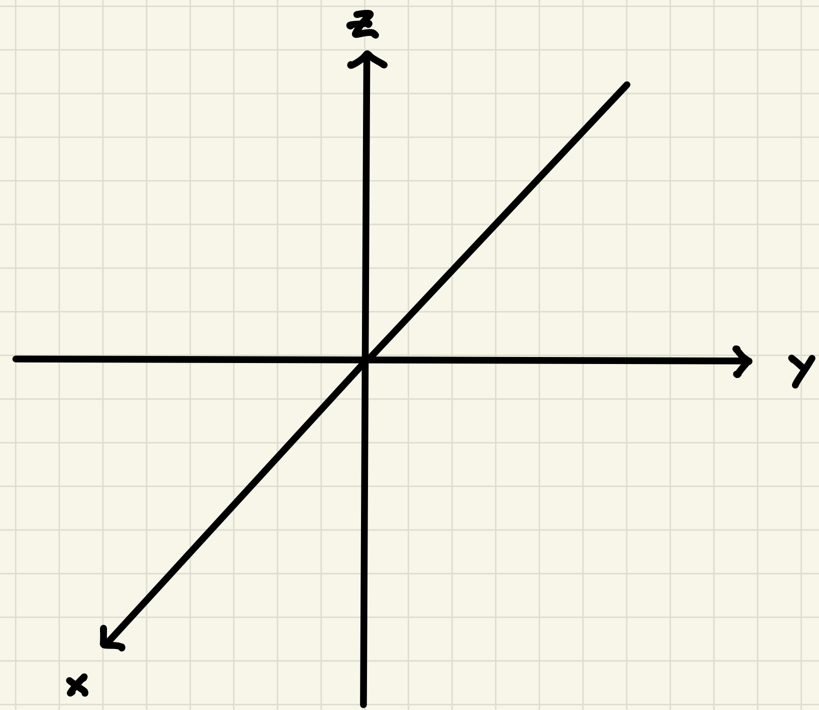
$|Q_1Q_2|$

$$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Graphing equations

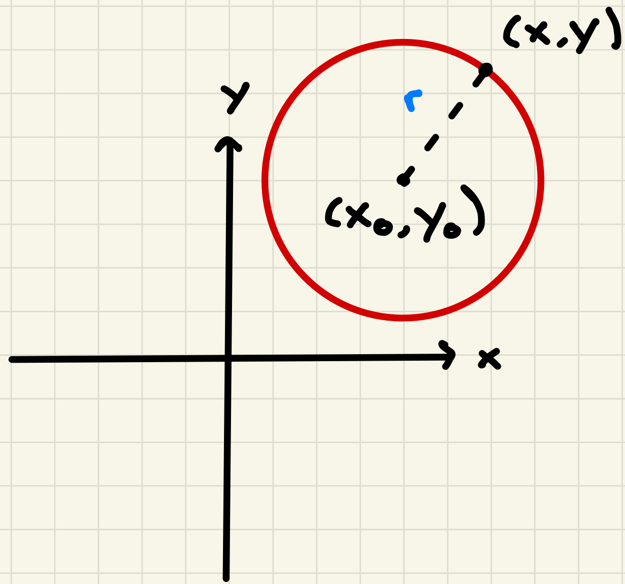
$$(x-x_0)^2+(y-y_0)^2=r^2$$



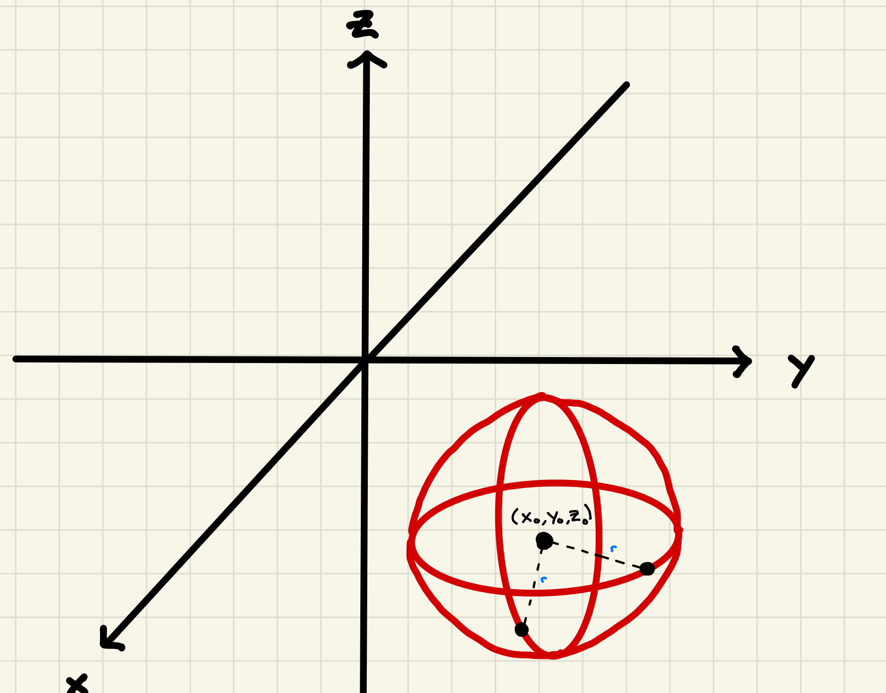


Graphing equations

$$(x-x_0)^2+(y-y_0)^2=r^2$$



$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$$



Equation of a sphere

A sphere of radius r and center (x_0, y_0, z_0)

is represented by the equation

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

If (x, y, z) satisfies this equation,

then it is at distance r away from

(x_0, y_0, z_0)

Equation of a sphere

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$$

Example: Show that $x^2+6x+y^2-4y+z^2-2z-100=0$ is the eq of a sphere and find the radius/center.

Idea: complete the square

$x^2+6x=(x+3)^2-9$

$y^2-4y=(y-2)^2-4$

$z^2-2z=(z-1)^2-1$

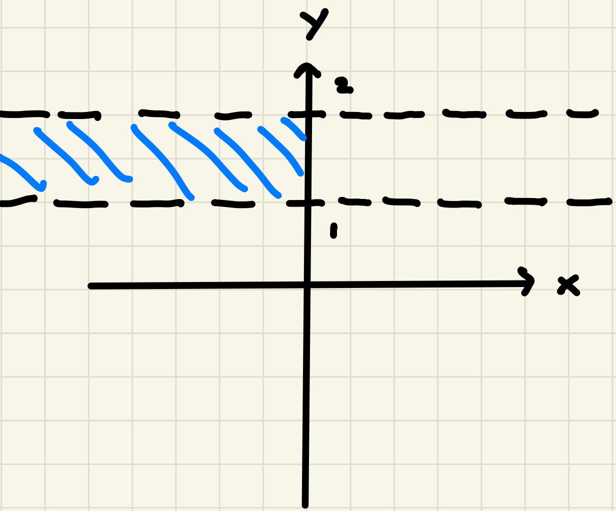
$(x+3)^2-9+(y-2)^2-4+(z-1)^2-1-100=0$

$\Rightarrow (x+3)^2+(y-2)^2+(z-1)^2=114$

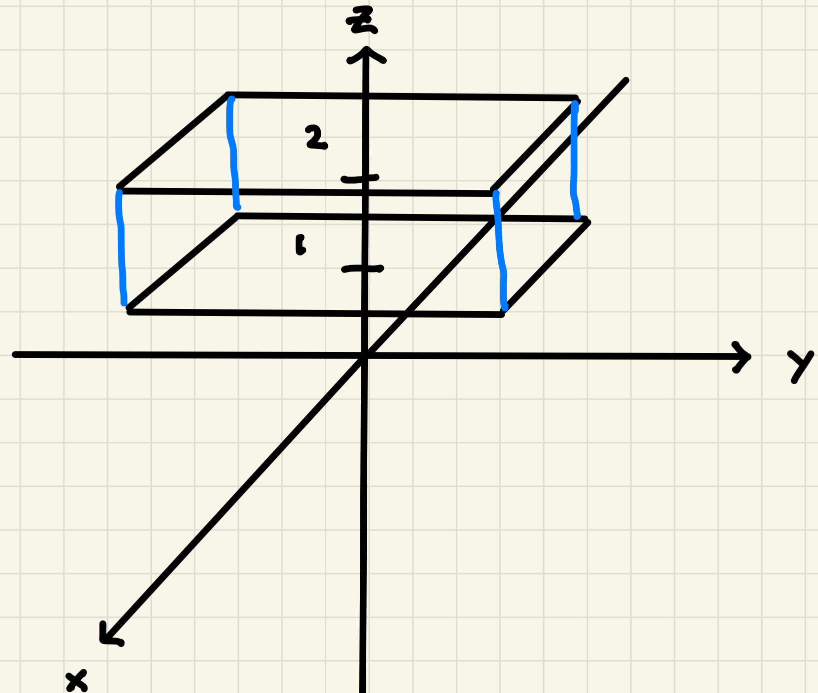
$\Rightarrow$ center is $(-3,2,1)$
radius is $\sqrt{114}$

Regions defined by multiple inequalities

$1 \le y \le 2, \quad x \le 0$



$1 \le z \le 2$



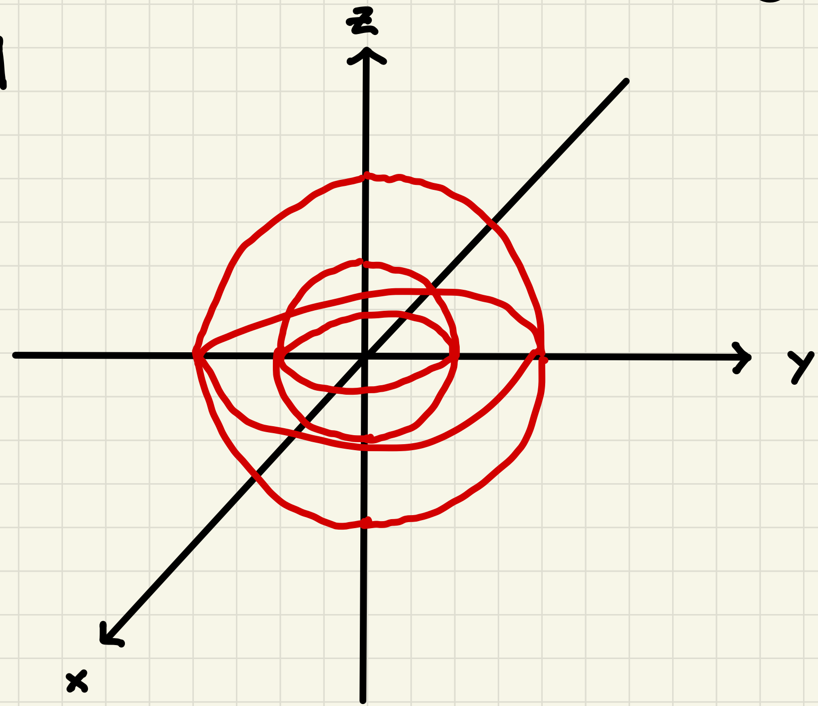
Moral: must satisfy all constraints

Regions defined by multiple inequalities

$4 \leq x^2 + y^2 + z^2 \leq 9$

$z \leq 0$

not yet

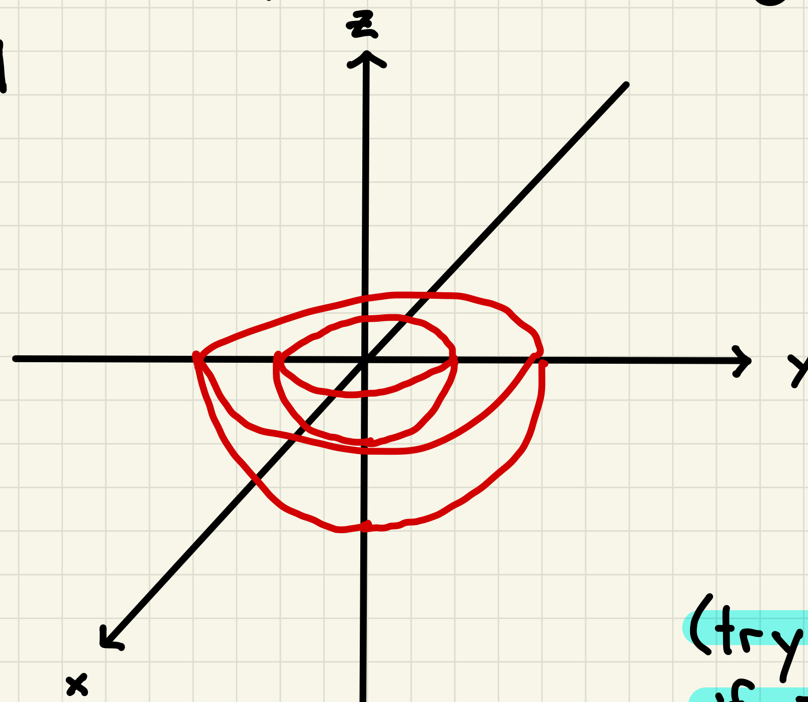


Moral: must satisfy all constraints

Regions defined by multiple inequalities

$$4 \le x^2 + y^2 + z^2 \le 9$$

$$z \le 0$$



(try to visualize if $z \le 0$ replaced by $z \le -1$)

Moral: must satisfy all constraints