Vector functions and space curves

Midterm 1 on Monday, Oct 26

See course website for more details

You will need to know how to use Gradescope!

Complete the practice assignment on the course website (under Announcements) beforehand!

Functions that you've seen before:

$f: \mathbb{R} \to \mathbb{R}$ ("scalar-valued function")

$g: A \to B$, where $A, B \subseteq \mathbb{R}$

For example, $g(x) = \sqrt{x}$ or $g(x) = \log(x)$

Today:

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\hat{i} + g(t)\hat{j} + h(t)\hat{k}$$

(vector-valued function)

f, g, and h are called the ==component functions of $\vec{r}$==

these are the kind of functions you're used to

Finding the domain of a vector-valued Function

just need to make sure EVERY component function can be evaluated (intersection of the domains of the component functions)

Example: $\vec{r}(t) = \left\langle \underbrace{\sqrt{t-2}}_{f(t)}, \underbrace{\frac{1}{t-4}}_{g(t)}, \underbrace{\log(5-t)}_{h(t)} \right\rangle$

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$f(t)$ $g(t)$ $h(t)$

$f(t)$ can only be evaluated for $t \in [2, \infty)$

$g(t)$ can only be evaluated for $t \in \mathbb{R} \setminus \{4\}$

$h(t)$ can only be evaluated for $t \in (-\infty, 5)$

Domain of $\vec{r}(t)$ is $[2,4) \cup (4,5)$

Taking the limit of a vector-valued function

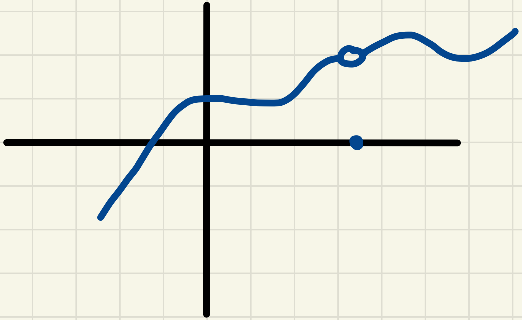
"where is it heading?"

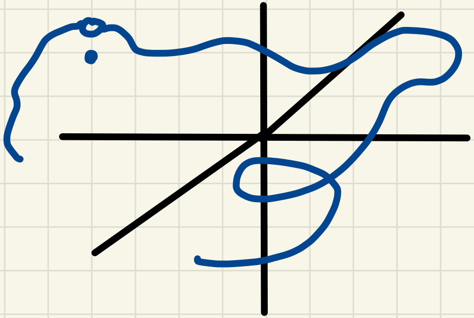
just need to take the limits of the components

If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$, then

$$\lim_{t \to a} \vec{r}(t) = \langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \rangle$$

assuming the limits of the component functions exist





Example : let $\vec{r}(t) = \left\langle t\cos(t), \dfrac{\sin(t)}{t}, (t+1)^2 \right\rangle$

find $\displaystyle\lim_{t \to 0} \vec{r}(t)$

Ans :

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find $\lim\limits_{t \to 0} \vec{r}(t)$

Ans: $\lim\limits_{t \to 0} t\cos(t) = 0 \cdot \cos(0) = 0 \cdot 1 = 0$

$\lim\limits_{t \to 0} \frac{\sin(t)}{t} = 1$ by L'Hôpital's rule

$\lim\limits_{t \to 0} (t+1)^2 = (0+1)^2 = 1$

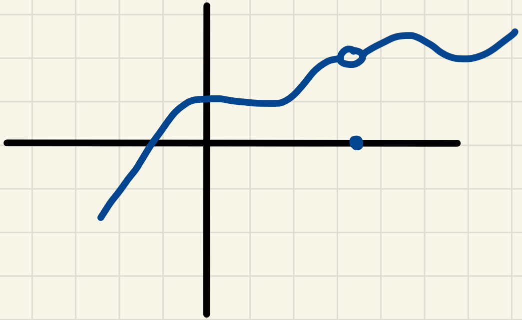
so $\lim\limits_{t \to 0} \vec{r}(t) = \langle 0, 1, 1 \rangle$

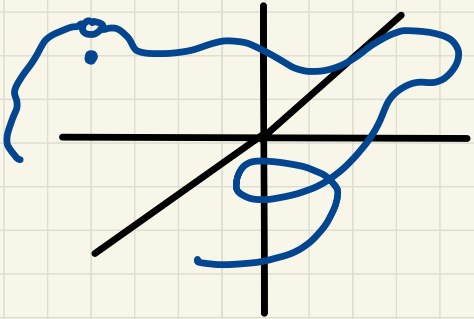
Continuity of a vector-valued Function

$\vec{r}(t)$ is continuous at $t=a$ if

$$\lim_{t \to a} \vec{r}(t) = \vec{r}(a)$$

"goes to where it's heading"





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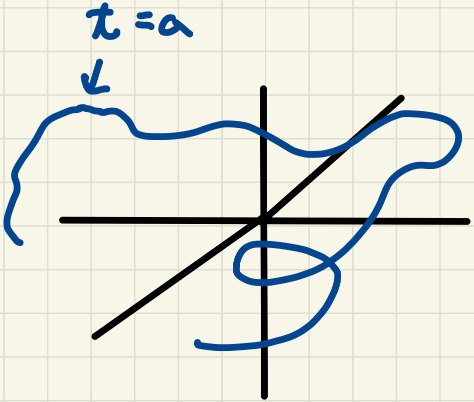


Continuity of a vector-valued function

$\vec{r}(t)$ is continuous at $t=a$ if

$\lim_{t \to a} \vec{r}(t) = \vec{r}(a)$ "goes to where it's heading"

Since $\lim_{t \to a} \vec{r}(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$,

$\vec{r}(t)$ is continuous at $t=a$ $\Longleftrightarrow$ its component functions are continuous at $t=a$

"the whole thing goes to where it's heading if and only if each piece does"

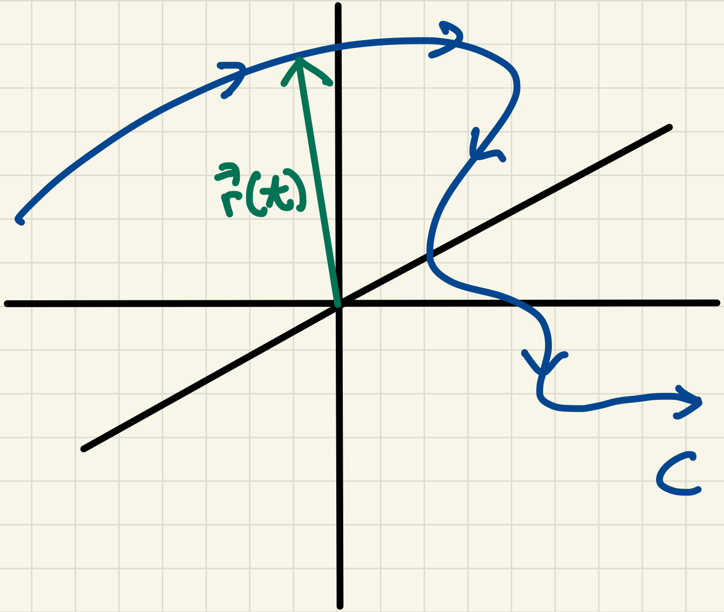
Space curves

Suppose that $\vec{r} : I \to \mathbb{R}^3$ is a vector-valued function with I an interval (e.g., $[0,2]$, $(0,1)$, $(3,\infty)$)

The range of this function (the set of all points (x,y,z) in space where $x = f(t)$, $y = g(t)$, and $z = h(t)$ as t varies through I) is called a space curve C

The component functions are called the parametric equations of C and t is called a parameter

We think of $\vec{r}(t)$ as giving the position of an object at time t (say an airplane) and the space curve as the path taken by the object



Example : describe the curve defined by

$$\vec{r}(t) = \langle 2t, -1+3t, 4+\pi t \rangle$$

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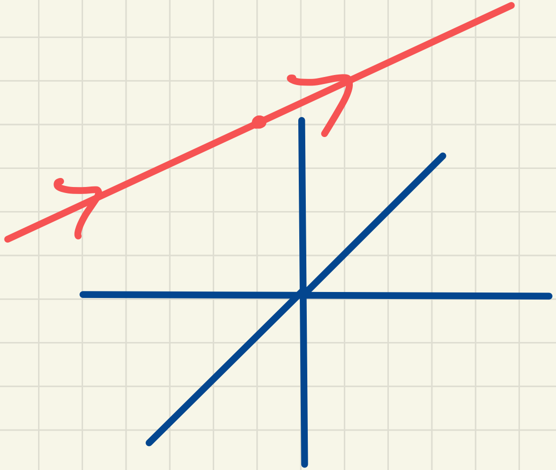
Ans :

$x = 2t$

$y = -1+3t$

$z = 4+\pi t$

$\Rightarrow$ line through the point $(0, -1, 4)$ with parallel vector $\langle 2, 3, \pi \rangle$



Example: sketch the curve whose vector eqn is

$$\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$$

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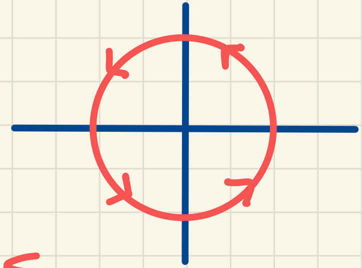
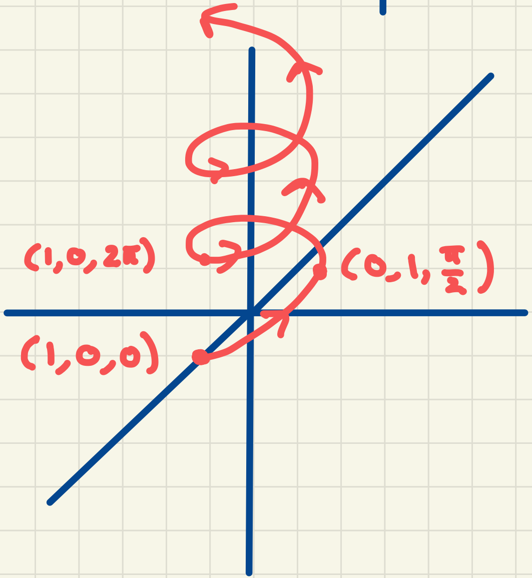
Ans : $\langle \cos(t), \sin(t) \rangle$ → circle



$\langle \cos(t), \sin(t), t \rangle$ → helix



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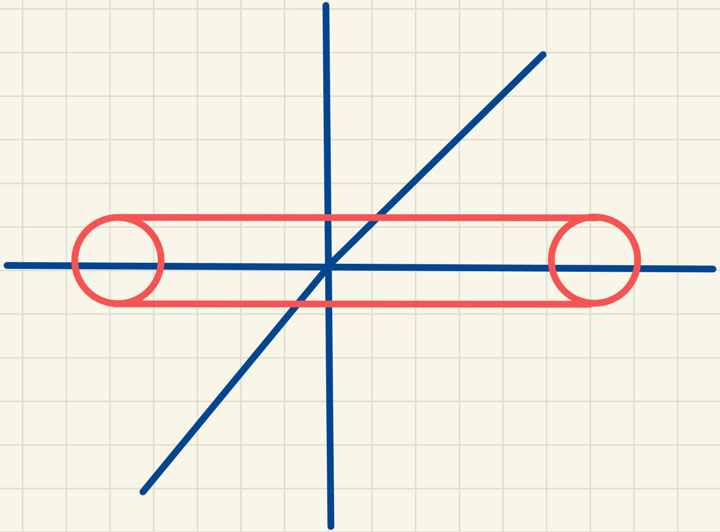

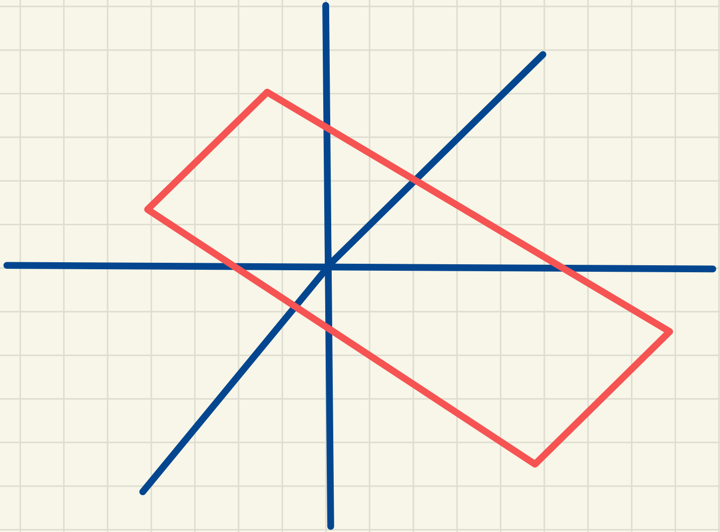

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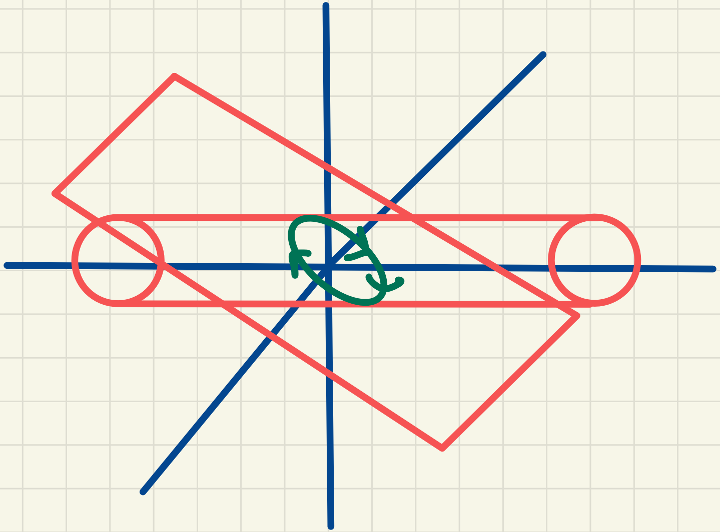

$x = \cos(t)$

$z = \sin(t)$

$y = -\sin(t)$

$0 \le t \le 2\pi$