MATH 180A - INTRODUCTION TO PROBABILITY
PRACTICE MIDTERM #1

FALL 2020

Name (Last, First): ____________________

Student ID: ____________________

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT.

1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space.

(a) Suppose that $A, B \in \mathcal{F}$ satisfy

$$\mathbb{P}(A) + \mathbb{P}(B) > 1.$$

Making no further assumptions on A and B, prove that $A \cap B \neq \emptyset$.

(b) Prove that A is independent from itself if and only if $\mathbb{P}(A) \in \{0, 1\}$.

2. Roll two fair dice repeatedly. If the sum is ≥ 10, then you win.

(a) What is the probability that you start by winning 3 times in a row?

(b) What is the probability that after rolling the pair of dice 5 times you win exactly 3 times?

(c) What is the probability that the first time you win is before the tenth roll (of the pair), but after the fifth?

3. A box contains 3 coins, two of which are fair and the third has probability 3/4 of coming up heads. A coin is chosen randomly from the box and tossed 3 times.

(a) What is the probability that all 3 tosses are heads?

(b) Given that all three tosses are heads, what is the probability that the biased coin was chosen?

4. Let X be a discrete random variable taking the values $\{1, 2, \ldots, n\}$ all with equal probability. Let Y be another discrete random variable taking values in $\{1, 2, \ldots, n\}$. Assume that X and Y are independent. Show that $\mathbb{P}(X = Y) = \frac{1}{n}$. (Hint: you do not need to know the distribution of Y to calculate this.)

5. Consider a point $P = (X, Y)$ chosen uniformly at random inside of the triangle in $\mathbb{R}^2$ that has vertices $(1, 0)$, $(0, 1)$, and $(0, 0)$. Let $Z = \max(X, Y)$ be the random variable defined as the maximum of the two coordinates of the point. For example, if $P = (\frac{1}{2}, \frac{1}{3})$, then $Z = \max(X, Y) = \frac{1}{2}$. Determine the cumulative distribution function of Z. Determine if Z is a continuous random variable, a discrete random variable, or neither. If continuous, determine the probability density function of Z. If discrete, determine the probability mass function of Z. If neither, explain why.

(Hint: Draw a picture.)