ANSWERS TO THE TRUE/FALSE QUESTIONS DO NOT NEED TO BE JUSTIFIED. A CORRECT ANSWER IS WORTH 5 POINTS, AN INCORRECT ANSWER IS WORTH 0 POINTS, AND A BLANK ANSWER IS WORTH 2 POINTS.

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT.

THIS EXAM WILL BE SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED.

THE EXAM CONSISTS OF 4 TRUE/FALSE QUESTIONS AND 3 LONGER FORMAT QUESTIONS. YOUR ANSWERS TO THE LONGER FORMAT QUESTIONS SHOULD BE CAREFULLY JUSTIFIED. YOU ARE ALLOWED TO USE RESULTS FROM THE TEXTBOOK, HOMEWORK, AND LECTURE, BUT THEY SHOULD BE CLEARLY REFERENCED. FOR EXAMPLE,

"We prove the statement by induction on ..."
1. (20 points) Label the following statements as true or false. Any ambiguous answer (for example, resembling a hybrid of T and F) will be treated as an incorrect answer.

(a) For any propositions $P$ and $Q$, the propositions $P \lor Q$ and $(\neg P) \land Q$ are logically equivalently. In other words,
$$ (P \lor Q) \iff ((\neg P) \land Q). $$

(b) For any subsets $A, B$ of a universal set $U$,
$$ (A \setminus B = \emptyset) \iff (B^c \setminus A^c = \emptyset). $$
(c) For any subsets $A, B, C$ of a universal set $U$, the following equality holds:

$((A \cap B) \cup C)^c = (A^c \cap C^c) \cup (B^c \cap C^c)$. 

(d) The following equality of sets holds:

$\{a + b + c | a, b, c \in \mathbb{Z}\} = \{m - n | m, n \in \mathbb{Z}\}$. 

2. (15 points) Let $a_0 = 0$, $a_1 = 1$, and $a_{n+2} = 6a_{n+1} - 9a_n$ for $n \geq 0$. Prove that $a_n = n3^{n-1}$ for all $n \geq 0$. 
3. (15 points) Fix a universal set $U$. Given sets $E, F \in \mathcal{P}(U)$, recall that their symmetric difference is defined by

$$E \Delta F = (E \setminus F) \cup (F \setminus E) = (E \cup F) \setminus (E \cap F).$$

Suppose that $A \in \mathcal{P}(U)$. Prove that there exists a unique set $B \in \mathcal{P}(U)$ such that

$$A^c = A \Delta B.$$
4. (15 pts) Let $n$ be an integer. Suppose that $2|n$ and $3|n$. Prove that $6|n$. 
(ADDITIONAL SPACE FOR WORK, CLEARLY INDICATE THE PROBLEM YOU ARE WORKING ON)