REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT.

THE EXAM CONSISTS OF 5 QUESTIONS. YOUR ANSWERS MUST BE CAREFULLY JUSTIFIED TO RECEIVE CREDIT.

IF YOU DO NOT ASSIGN THE PAGES OF YOUR WORK TO THE QUESTIONS OF THE EXAM IN YOUR UPLOAD TO GRADESCOPE, YOU WILL LOSE 2 POINTS.

WRITE YOUR NAME AND STUDENT ID ON THE FIRST PAGE OF YOUR UPLOAD.
Recall that you do not need to copy down the questions.

1. (7 points) Let

\[ f(x) = e^{e^{\sqrt{x}}} = e^{(e^{(\sqrt{x})})}. \]

Find the derivative \( f'(x) \). (Note: I have included the parentheses as a hint. Be careful, the function \( f(x) \neq e^{e^{\sqrt{x}}} \). Simply writing down an answer without any intermediate steps will receive 0 points. You can leave your answer unsimplified as long as there are no derivatives left to calculate.)

**Proof.** We compute

\[
\begin{align*}
  f'(x) &= e^{e^{\sqrt{x}}} \frac{d}{dx} \left[ e^{\sqrt{x}} \right] \\
         &= e^{e^{\sqrt{x}}} e^{\sqrt{x}} \frac{d}{dx} \left[ \sqrt{x} \right] \\
         &= e^{e^{\sqrt{x}}} e^{\sqrt{x}} \frac{1}{2\sqrt{x}}.
\end{align*}
\]

\[ \square \]
2. (7 points) Let
\[ g(x) = \frac{x^{2021} e^x}{\sin(x)}. \]
Find the derivative \( g'(x) \). (Note: simply writing down an answer without any intermediate steps will receive 0 points. You can leave your answer unsimplified as long as there are no derivatives left to calculate.)

*Proof.* We compute
\[
g'(x) = \frac{\sin(x) \frac{d}{dx} [x^{2021} e^x] - x^{2021} e^x \frac{d}{dx} [\sin(x)]}{(\sin(x))^2}
\]
\[
= \frac{\sin(x)(2021x^{2020} e^x + x^{2021} e^x) - x^{2021} e^x \cos(x)}{\sin^2(x)}.
\]
\[ \square \]
3. (7 points) A ball is thrown straight up into the air from the ground. Its height at time $t$ is given by the function $h(t) = 10t - 4.9t^2$, where $t$ is measured in seconds and $h(t)$ is measured in meters.

(a) (3 points) Find the velocity function $v(t)$ of the ball.

(b) (4 points) How many seconds does it take for the ball to return to the ground?

Proof of (a).

$$v(t) = h'(t) = 10 - 9.8t.$$ □

Proof of (b). We need to solve $h(t) = 0$ for some value of $t \neq 0$. So,

$$h(t) = 0 \iff 10t - 4.9t^2 = 0 \iff t(10 - 4.9t) = 0.$$ Since we want $t \neq 0$, this means $10 - 4.9t = 0$ or $t = \frac{10}{4.9}$. □
4. (12 points)

Assume that the orange curve is the graph of the function $g'(x)$ (read this carefully: this is the graph of $g'(x)$, the derivative of $g(x)$).

(a) (6 points) Where is the function $g(x)$ increasing? Where is the function $g(x)$ decreasing?

(b) (6 points) Where is the function $g(x)$ concave upward? Where is the function $g(x)$ concave downward?

Proof of (a). $g(x)$ is increasing where $g'(x) > 0$, which is for $x \in (-6, 7)$. $g(x)$ is decreasing where $g'(x) < 0$, which is for $x \in (-\infty, -6) \cup (7, \infty)$. □

Proof of (b). $g(x)$ is concave upward where $g''(x) > 0$, which is where $g'(x)$ is increasing. Thus, $g(x)$ is concave upward for $x \in (-\infty, -3) \cup (0, 4)$. Similarly, $g(x)$ is concave downward where $g''(x) < 0$, which is where $g'(x)$ is decreasing. Thus, $g(x)$ is concave downward for $x \in (-3, 0) \cup (4, \infty)$. □
5. (7 points) Compute the limit

\[ \lim_{{x \to \infty}} \frac{2x^2 + \sqrt{x}}{3x^2 + 1} \]

(Note: simply writing down an answer without any intermediate steps will receive 0 points.)

**Proof.**

\[ \frac{2x^2 + \sqrt{x}}{3x^2 + 1} = \frac{\frac{1}{x^2}(2x^2 + \sqrt{x})}{\frac{1}{x^2}(3x^2 + 1)} = \frac{2 + \frac{1}{x^{3/2}}}{3 + \frac{1}{x^2}}. \]

So,

\[ \lim_{{x \to \infty}} \frac{2x^2 + \sqrt{x}}{3x^2 + 1} = \lim_{{x \to \infty}} \frac{2 + \frac{1}{x^{3/2}}}{3 + \frac{1}{x^2}} = \frac{\lim_{{x \to \infty}} 2 + \frac{1}{x^{3/2}}}{\lim_{{x \to \infty}} 3 + \frac{1}{x^2}} = \frac{2}{3}. \]