

The image of 3 different circles under $f(z) = z^2 + z$.

Let $h(z) = z + z^{-1}$ and note that $h(z) = 0$ iff $z^2 + 1 = 0$, i.e. iff $z = \pm i$.

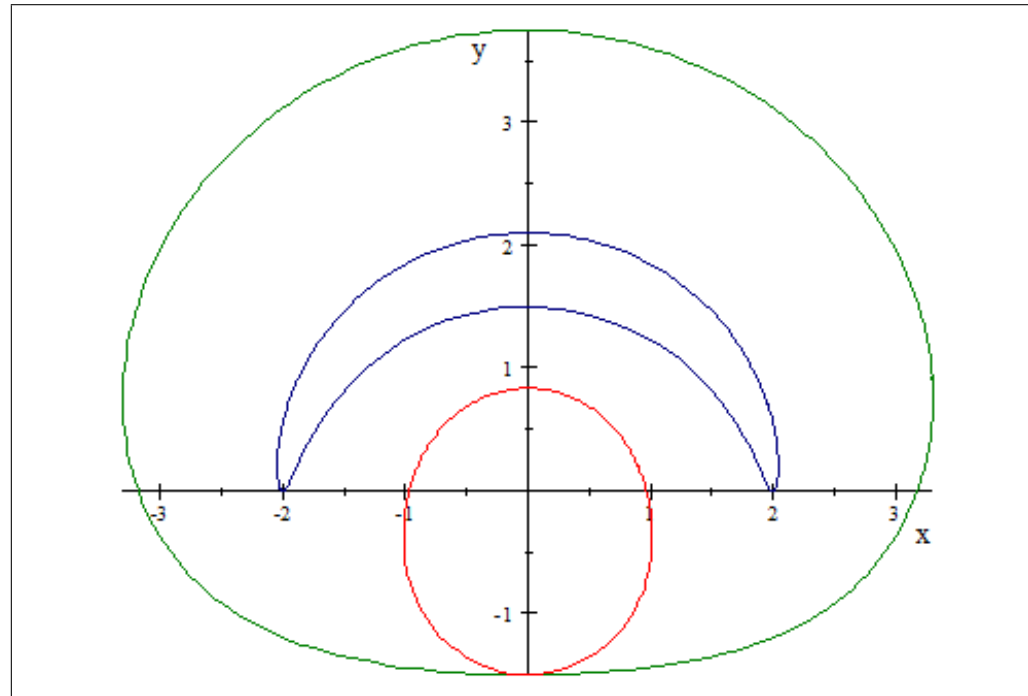


FIGURE 1. Here $h(z) = z + z^{-1}$ and these are plots of $h(i + 3e^{i\theta})$ in green, $h(i + 1.5e^{i\theta})$ in blue, and $h(i + \frac{1}{2}e^{i\theta})$ in red.

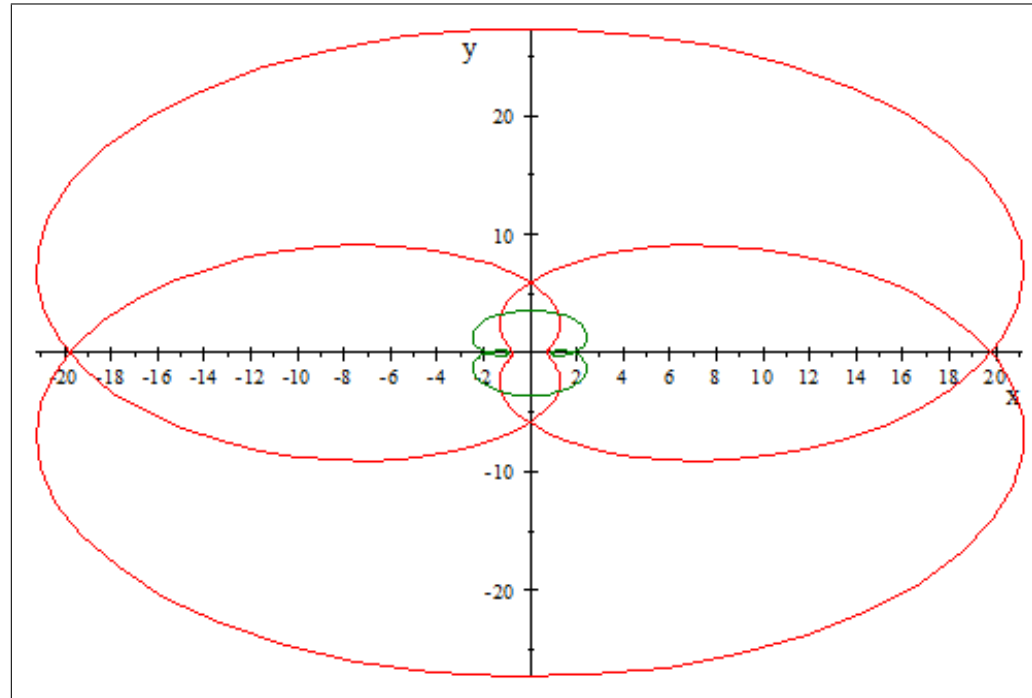


FIGURE 2. Plots of $\sin(4e^{i\theta})$ in red and $\sin(2e^{i\theta})$ in green with winding number 3 and 1 respectively.

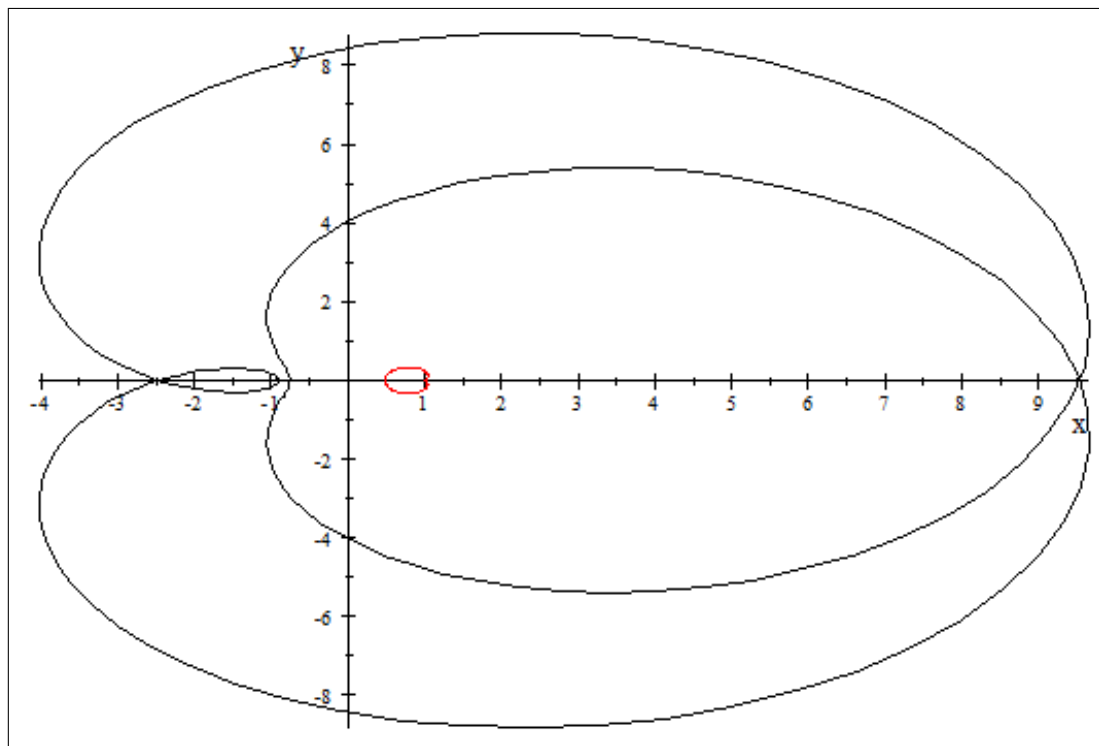


FIGURE 3. Plots of $\sin(1 + 3e^{i\theta})$ in black and $\sin(1 + \frac{1}{2}e^{i\theta})$ in red with winding numbers about 0 being 2 and 0 respectively.

$$g(z) = z \left(1 + \frac{1}{10} \left(z - \frac{1}{3} \right)^3 \left(z - \frac{1}{2} \right)^2 \right)$$

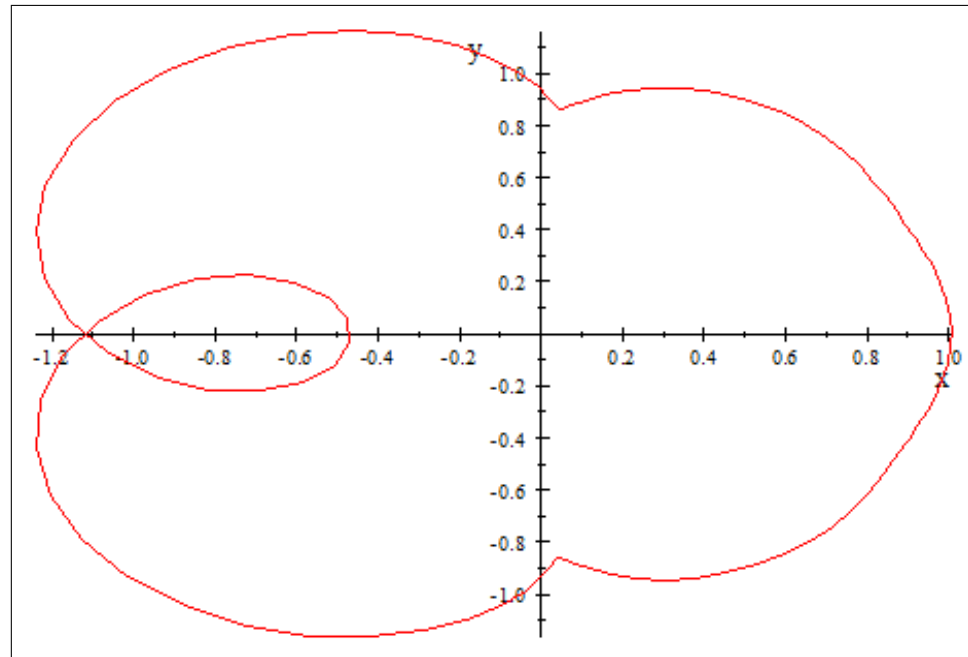


FIGURE 4. Plot of $g(e^{i\theta})$ for $0 \leq \theta \leq 2\pi$ in reference to the inverse function theorem.

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$$f(z) = z \left(1 + \frac{1}{10} \left(z - \frac{1}{3} \right)^3 \left(z - \frac{i}{2} \right)^2 \right)$$

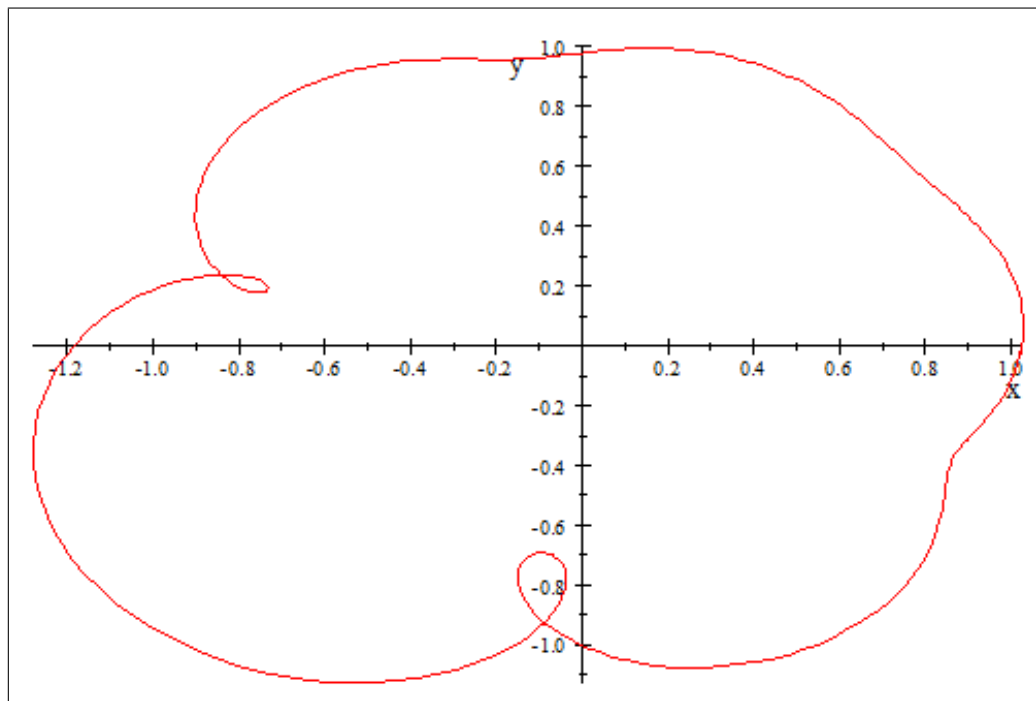


FIGURE 5. Plot of $f(e^{i\theta})$ for $0 \leq \theta \leq 2\pi$ in reference to the inverse function theorem.