Part I

140B Homework Exercises (Winter 2013)

- The problems below either come from the lecture notes or from Rudin's "Principles of Real Analysis," 3rd ed.
- Although pictures are not typically provided in these solutions, I would highly advise that the reader draw appropriate pictures whenever possible. Many of the proofs to follow are really a transcription of a very natural picture proof to a formal written proof.

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HomeWork #1 Due Friday, January 11, 2013)

Exercise 1.1. Prove item 2. of Theorem 9.7. If $\{C_\alpha\}_{\alpha\in I}$ is a collection of closed subsets of X, then $\cap_{\alpha\in I}C_\alpha$ is closed in X.

Exercise 1.2. Give an example of a collection of closed subsets, $\{A_n\}_{n=1}^\infty$, of $\mathbb{C}$ such that $\cup_{n=1}^\infty A_n$ is not closed.

Exercise 1.3. Prove Corollary 9.8 from the text. That is; if (X,d) be a metric space, then the collection of open subsets, τ_d, of X satisfy;

1. X and $\emptyset$ are in τ_d.
2. τ_d is closed under taking arbitrary unions. i.e. if $\{U_\alpha\}_{\alpha\in I}$ is a collection of open sets then $\cup_{\alpha\in I}U_\alpha$ is open..
3. τ_d is closed under taking finite intersections, i.e. if U and V are open sets then then $U\cap V$ is open as well.

Exercise 1.4. Let U be a subset of a metric space (X,d). Show the following are equivalent;

1. U is open,
2. for all $z\in U$ there exists $\rho>0$ such that $B_z(\rho)\subset U$.
3. U can be written as a union of open balls.

Exercise 1.5. Let (X,d) be a metric space and $\{x_1,\dots,x_n\}$ be a finite subset of X. Show $X\setminus\{x_1,\dots,x_n\}$ is an open subset.

Exercise 1.6. Let (X,d) be a complete metric space. Let $A\subset X$ be a subset of X viewed as a metric space using $d|_{A\times A}$. Show that $(A,d|_{A\times A})$ is complete iff A is a closed subset of X.

Exercise 1.7. If A is a non-empty subset of X, then $d_A=d_{\bar{A}}$.

Exercise 1.8. Given $A\subset X$, show $\bar{A}$ is a closed set and in fact

$$\bar{A}=\cap\{F: A\subset F\subset X \text{ with } F \text{ closed}\}. \tag{1.1}$$

That is to say $\bar{A}$ is the smallest closed set containing A.