Math 180B, Spring 2020 Homework

0. Homework 0

This assignment will familiarize you with Gradescope, the software used in this course for grading homework and exams. Complete the following steps.

(1) Write up a solution for Question 0.1 below. [Try to work out Question 0.2 as a review but do not turn in this question.]
(2) Make sure to include your name, student ID (AXXXXXXXX), and the name of your professor in the top right-hand corner of your assignment. For longer assignments later in the quarter, this only needs to be done on the first page.
(3) Create an account with Gradescope, linked to your `@ucsd.edu` address. If you already have a Gradescope account linked to your `@ucsd.edu` address, then you do not need to create another account.
(4) Read the guide for submitting homework on Gradescope.
(5) Log into your account, find the course Gradescope page, select the correct assignment, and upload your scanned assignment following the guide in the previous step. If you cannot find the course Gradescope page, then you may need a course entry code 9DXBNR.

Question 0.1. Let $\{\lambda_n\}_{n=1}^{\infty}$ be a sequence of real numbers such that $\lambda = \lim_{n\to\infty} \lambda_n$ exists in $\mathbb{R}$. Show for each $k \in \mathbb{N}_0$ that

$$\lim_{n\to\infty} (1 - \lambda_n/n)^{n-k} = e^{-\lambda}.$$

Hint: you might use $\ln(1+x) = x + O\left(x^2\right)$ for $|x|$ small.]

Question 0.2 (Some Discrete Distributions)**. Look at but do not hand in.** Let $p \in (0,1]$ and $\lambda > 0$. In the four parts below, the distribution of N will be described. You should work out the generating function,

$$G_N(z) := \mathbb{E}\left[z^N\right] = \sum_{n=0}^{\infty} \mathbb{P}(N = n)\, z^n \text{ for } |z| \le 1$$

and then use it to verify the given formulas for $\mathbb{E}N$ and $\operatorname{Var}(N)$.

(1) Bernoulli(p) : $\mathbb{P}(N=1) = p$ and $\mathbb{P}(N=0) = 1-p$. You should find
$$\mathbb{E}N = p \text{ and } \operatorname{Var}(N) = p - p^2.$$
(2) Binomial$(n,p) = \operatorname{Bin}(n,p)$:
$$\mathbb{P}(N=k) = \binom{n}{k} p^k (1-p)^{n-k} \text{ for } k = 0, 1, \dots, n,$$
i.e. $\mathbb{P}(N=k)$ is the probability of k successes in a sequence of n independent yes/no experiments with probability of success being p.) You should find
$$\mathbb{E}N = np \text{ and } \operatorname{Var}(N) = n\left(p - p^2\right).$$
(3) Geo(p) : $\mathbb{P}(N=k) = p(1-p)^{k-1}$ for $k \in \mathbb{N}$ is the **geometric distribution.** [$\mathbb{P}(N=k)$ is the probability that the k^{th} – trial is the first time of success out a sequence of independent trials with probability of success being p.] You should find
$$\mathbb{E}N = \frac{1}{p} \text{ and } \operatorname{Var}(N) = \frac{1-p}{p^2}.$$
(4) Poisson(λ) : $\mathbb{P}(N=k) = \frac{\lambda^k}{k!} e^{-\lambda}$ for all $k \in \mathbb{N}_0 = \{0, 1, 2, \dots\}$, see Proposition **??** below for some context. You should find
$$\mathbb{E}N = \lambda = \operatorname{Var}(N).$$