Math 180B, Spring 2020 Homeworks

1. HOMEWORK 1

Question 1.1. A fair coin is tossed 20 times. Let X be the number of heads thrown in the first 10 tosses, and let Y be the number of heads tossed in the last 10 tosses. Find the conditional probability that $X = 6$, given that $X + Y = 10$.

Question 1.2. Let X_1 and X_2 be independent Poisson random variables with parameters λ_1 and λ_2. Show that for every $n \geq 1$, the conditional distribution of X_1, given $X_1 + X_2 = n$, is binomial, and find the parameters of this binomial distribution.

Question 1.3 (See Durrett, #8, p. 213)**.** Suppose that X and Y are two integrable discrete random variables such that

$$\mathbb{E}[X|Y] = 18 - \frac{3}{5}Y \text{ and } \mathbb{E}[Y|X] = 10 - \frac{1}{3}X.$$

Find $\mathbb{E}X$ and $\mathbb{E}Y$.

Question 1.4. An item is selected randomly from a collection labeled $1, 2, \ldots, n$. Denote its label by Y. Now select an integer X uniformly at random from $\{1, \ldots, Y\}$. Find:
  (a) $\mathbb{E}(X)$;
  (b) $\mathbb{E}(X^2)$;
  (c) $\mathrm{Var}(X)$;
  (d) $\mathbb{P}(Y + X = 3)$.
  You may freely use:

$$\sum_{k=1}^{n} k = \frac{1}{2}n(n+1) \text{ and } \sum_{k=1}^{n} k^2 = \frac{1}{6}n(2n+1)(n+1) = \frac{2n^3 + 3n^2 + n}{6}.$$

Question 1.5. Pinsky and Taylor **Problem** 2.1.2. Please write your answer in terms of the "*harmonic number*" function defined by,

$$H(y) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{y} \text{ for } y \in \mathbb{N}.$$

Question 1.6. Pinsky and Taylor **Problem** 2.1.6.