Math 20E Practice Test #1 (Driver)

Notation: Throughout this test $\mathbf{R} = (x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ will denote a position vector and $\mathbf{F}(\mathbf{R}) = F_1(\mathbf{R})\mathbf{i} + F_2(\mathbf{R})\mathbf{j} + F_3(\mathbf{R})\mathbf{k}$ will denote a vector field.

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1. Captain Ralph is in trouble near the sunny side of Mercury. The temperature of the ship's hull when he is at location (x, y, z) will be given by $T(x, y, z) = \exp\left(-x^2 - 2y^2 - 3z^2\right)$, where x, y, and z are measured in meters. He is currently at $(1, 1, 1)$.
   (a) In what direction should he proceed in order to decrease the temperature most rapidly?
   (b) If the ship travels at e^8 meters per second, how fast will be the temperature decrease if he proceeds in that direction (degrees per second)?
   (c) What is the rate of change of the temperature at $t = 0$ if the ship moves along a curve $\mathbf{R}(t) \in \mathbb{R}$ such that $\mathbf{R}(0) = (1, 1, 1)$, and $\dot{\mathbf{R}}(0) = \frac{d}{dt}\mathbf{R}(t)|_{t=0} = (-1, 2, -3)$.

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2. Find tangent plane to the surface S described by $z = \frac{1}{x^2+y^2}$ at $(1, 1, \frac{1}{2})$.

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3. Suppose that a small weather balloon is placed at $t = 0$ at location $(1, 1, 1)$ in a wind velocity field described by $\mathbf{F}(x, y, z) = (y + 1, 2, \frac{1}{2z})$. Show $\mathbf{R}(t) = \left((t + 1)^2, 1 + 2t, \sqrt{t + 1}\right)$ (for $t > 0$) describes the position of the balloon at later times.

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4. Given $\mathbf{F} = (x + xz^2)\mathbf{i} + xy\mathbf{j} + yz\mathbf{k}$ evaluate:
   (a) $\nabla \cdot \mathbf{F}$
   (b) $\nabla \times \mathbf{F}$.

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5. Compute the divergence and curl of the following vector fields.

$$V(x, y, z) = -y\mathbf{i} + x\mathbf{j}$$
$$W(x, y, z) = \frac{V(x, y, z)}{(x^2 + y^2)^{1/2}}$$
$$Y(x, y, z) = \frac{V(x, y, z)}{(x^2 + y^2)}.$$

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6. Compute $\int_C \mathbf{F} \cdot dR$ where $\mathbf{F}(x, y, z) = (xy, yz, xz)$ and C is the straight line path from $(0, 0, 0)$ to $(1, 1, 1)$.

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7. Suppose $\mathbf{F} = (F_1(x, y), F_2(x, y), 0)$ is a vector field which at $z = 0$ is pictured in Figure 1 below.

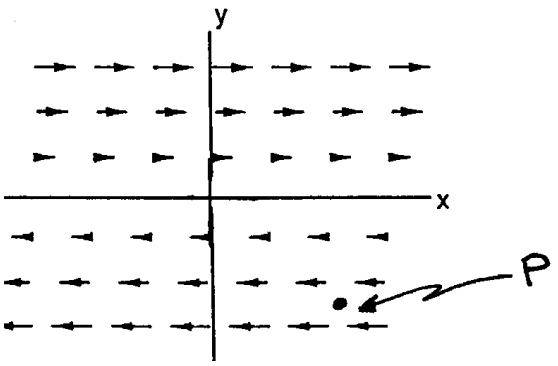

FIGURE 1. Graphical representation of $\mathbf{F}$ when $z = 0$.

(a) Say whether $\nabla \cdot \mathbf{F}(P)$ is positive, zero, or negative.
(b) Say whether $\mathbf{k} \cdot \nabla \times \mathbf{F}(P)$ is positive, zero, or negative.
Give reasons for your answer.

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8. At what angle does the line $2x = y = z$ intersect the ellipsoid

$$2x^2 + 3y^2 + z^2 = 18.$$