

Math 20E Practice Test #2 (Driver)

Instructions: Please put your name and section number or time on your blue book! You should do all of the problems. Please do not use calculators or books. You are allowed one 3×5 index card of notes. Good luck.

- (1) Determine which of the following vector fields below are conservative. For the conservative vector fields find a function f such ∇f is the given vector field.
- (a) $\mathbf{F} = 2x e^{x^2+y^2} \mathbf{i} + (2y e^{x^2+y^2} + y) \mathbf{j} + e^z \mathbf{k}$
 - (b) $\mathbf{G} = xy \mathbf{i} + y \mathbf{j} + xy \mathbf{k}$
 - (c) $\mathbf{H} = 2x \mathbf{i} + 2y \mathbf{j} + 2z \mathbf{k}$
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- (2) Compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{R}$ where \mathbf{F} is the field in 1a and C is the curve:

$$x = t^2, y = \sin 2\pi t \text{ and } z = \cos 2\pi t \text{ with } t : 0 \rightarrow 1.$$

- (3) Verify the divergence theorem for the field $\mathbf{F} = (xz, yz, 3z^2)$ on the region E bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 1$. (Both integrals should be equal to $\frac{8}{3}\pi$.)
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- (4) Find the surface integral of $\mathbf{F} = x \mathbf{i} + y \mathbf{j}$ over the surface of the cylinder of radius 2, height 3 centered on the z -axis with base in the xy -plane.
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- (5) Compute the integral $\iint_D xye^{(x-y)}(y+x)dxdy$ where $D := \{(x, y) : 1 \leq xy \leq 2 \text{ and } -1 \leq y - x \leq 1\}$.
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- (6) Suppose that a surface Σ is parametrized as $\mathbf{R}(u, v) = (uv, e^u, v + u)$ for $(u, v) \in D$ (D is some region in \mathbb{R}^2), then

$$\iint_{\Sigma} f dS = \iint_D f(uv, e^u, v + u) J(u, v) du dv$$

for some function J . Find J .