

## Math 240 Topics

### Primary Topics:

(These topics should be covered as qual material. The choice of order is subject to the instructors preferences.)

1. Introduction to sigma - algebras, measurable functions, and measures.
2. The structure of measurable sets and functions.
3. Construction of the integral from a measure
4. General properties of the integral (Fatou's lemma, monotone convergence, Lebesgue dominated convergence, Tchebychev inequality, Jensen's inequality)
5. Product measures and the Fubini-Tonelli theorems
6. Lebesgue Measure on  $\mathbf{R}^d$  and the change of variables theorem
7.  $L^p$  spaces, Holder inequality, the dual of  $L^p$  spaces.
8. Radon-Nikodym Theorem for Positive measures.
9. Signed and complex measures, Hahn decomposition, Jordan decomposition and the Radon - Nikodym theorem.
10. Differentiation of measures on  $\mathbf{R}^d$  and the fundamental theorem of calculus.
11. Point set topology: connectedness, compactness, countability axioms, along with the Baire category theorem.
12. More point set topology: Locally compact Hausdorff spaces, Uryshon's Lemma, Tietze Extension Theorem, Partitions of Unity, Alexandrov's compactification, Uryshon's metrization theorem.
13. Density and approximation theorems including: the Stone Weierstrass theorem and the use of convolution to smooth functions.
14. Hilbert space theory including projection theorems and orthonormal bases.
15. Fourier series and integrals, Plancherel theorem.
16. Basic theory of bounded linear operators and functionals.
17. Basic Banach space results including: Banach Steinhaus Theorem (uniform boundedness principle), Hahn Banach Theorem, Open mapping theorem and the closed graph theorem.
18. weak, weak\* topologies and Alaoglu's theorem.
19. Introduction to the Fourier Transform.

### Secondary Topics:

(These topics or others to be covered as the instructor sees fit.)

1. An introduction to Distribution theory with perhaps some elliptic regularity theory
2. More on the Fourier Transform and its properties with basic applications to PDE and Sobolev spaces.
3. Riesz Representation theorem for measures and applications to construction of measures..
4. The Spectral Theorem for bounded self-adjoint operators on a Hilbert space
5. Unbounded operators and the Spectral Theorem for self-adjoint operators.
6. Some calculus on Banach spaces and the Inverse and implicit function theorems with applications to ordinary differential equations.