# (11/10/2010)

Second midterm on Wednesday, 11/10/2010.

## 21.1 Study Guide for Math 120A Exam 2 (What you should know)

1. $e^z = e^x(\cos y + i\sin y)$ and $|e^z| = e^x = e^{\operatorname{Re} z} \le e^{|z|}$.
2. $\arg(z) = \left\{\theta \in \mathbb{R} : z = |z| e^{i\theta}\right\}$ and $\operatorname{Arg}(z) = \theta$ if $-\pi < \theta \le \pi$ and $z = |z| e^{i\theta}$. Notice that $z = |z| e^{i\arg(z)}$
3. $\log z = \ln|z| + i\arg z$ and its branches. I will denote a typical branch of log by $\ell$. Recall that $\ell'(z) = 1/z$ for all branches, $\ell$, of log.
4. $z^{1/n} = \sqrt[n]{|z|}e^{i\frac{\arg(z)}{n}}$ – a branch of $z^{1/n}$ is $z_\ell^{1/n} := e^{\frac{1}{n}l(z)}$.
5. More generally if $a \in \mathbb{C}$ and $\ell$ is a branch of log, the we define $z_\ell^a := e^{a\ell(z)}$ and we have
$$\frac{d}{dz} z_\ell^a = a z_\ell^{a-1}.$$
6. Be familiar with the following analytic functions;
   a) $\sin(z) := \frac{e^{iz} - e^{-iz}}{2i} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1}$
   b) $\cos(z) := \frac{e^{iz} + e^{-iz}}{2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} z^{2n}$
   c) $\sinh(z) := \frac{e^z - e^{-z}}{2} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} z^{2n+1}$
   d) $\cosh(z) := \frac{e^z + e^{-z}}{2} = \sum_{n=0}^{\infty} \frac{1}{(2n)!} z^{2n}$
   e) $\tan(z) = \frac{\sin(z)}{\cos(z)} = -i\frac{e^{iz} - e^{-iz}}{e^{iz} + e^{-iz}}$
   f) $\tanh(z) = \frac{\sinh(z)}{\cosh(z)} = \frac{e^z - e^{-z}}{e^z + e^{-z}}$
7. You should have some familiarity with some of the inverse trig functions as well.
8. Recall that analytic functions satisfy the chain rules;
$$\frac{d}{dz} f(g(z)) = f'(g(z))\, g'(z)$$
and
$$\frac{d}{dt} f(z(t)) = f'(z(t))\, \dot{z}(t).$$
9. It is sometimes useful to know that derivatives of inverse functions can be found using the "converse" to the chain rule covered in class.
10. Integration:
$$\int_a^b z(t)\, dt := \int_a^b x(t)\, dt + i\int_a^b y(t)\, dt.$$
All of the usual integration rules hold, like the fundamental theorem of calculus, linearity and integration by parts.
11. Be able to use the Cauchy Riemann equations to check that a function is analytic.
12. Know how to find harmonic conjugates.
13. Be able to parametrize simple contours.
14. Be able to compute contour integrals by parametrizing the contour and then evaluating the integral using
$$\int_C f(z)\, dz = \int_a^b f(z(t))\, \dot{z}(t)\, dt$$
where $C$ is the contour $z = z(t)$ with $a \le t \le b$.
15. Be able to estimate contour integrals using
$$\left|\int_C f(z)\, dz\right| \le \max_{z\in C} |f(z)| \cdot \text{length}(C).$$
16. Be able to compute contour integrals using the fundamental theorem of calculus: **if** $f$ is analytic on a neighborhood of a contour $C$, then
$$\int_C f'(z)\, dz = f(C_{\text{end}}) - f(C_{\text{begin}}).$$
17. General knowledge of material from the start of the course. For example you should recall that $z^{1/n} = \sqrt[n]{|z|}e^{i\frac{\arg(z)}{n}}$ where $\arg(z) = \left\{\theta \in \mathbb{R} : z = |z| e^{i\theta}\right\}$.