Let $n \in \mathbb{Z}$.

Let $P(n)$ be the statement: There exists $q \in \mathbb{Z}$ such that $n = 5q + 3$.

Let $Q(n)$ be the statement: There exists $p \in \mathbb{Z}$ such that $n^2 = 5p + 4$.

**Exercise.** Prove that $P(n)$ implies $Q(n)$.

**Solution.** Suppose $n \in \mathbb{N}$ is such that $P(n)$ is true. Then there exists $q \in \mathbb{Z}$ such that $n = 5q + 3$. This implies

$$n^2 = (5q + 3)^2 = 25q^2 + 30q + 9 = 5(5q^2 + 6q + 1) + 4.$$  

Let $p = 5q^2 + 6q + 1$. Then $p \in \mathbb{Z}$ and $n^2 = 5p + 4$. Hence $Q(n)$ is true. □